§3.10 7. a) Given: a man 6 ft tall walks away from a street light mounted on a 15 ft tall pole at a rate of 5 ft/s. \( t = \text{time} \) \( x = \text{distance} \) from man to pole, \( \frac{dx}{dt} = 5 \text{ft/s} \).

b) unknown: rate at which tip of shadow is moving when he is 40 ft from pole, \( y = \text{distance from man to tip of his shadow}, \) we want \( \frac{dy}{dt} \) when \( x = 40 \text{ft} \).

c) By similar triangles, \( \frac{15}{6} = \frac{x+y}{y} \Rightarrow 15y = 6x + 6y \Rightarrow 9y = 6x \Rightarrow y = \frac{2}{3}x \)

d) The tip of the shadow moves at a rate of \( \frac{dy}{dx} \left( x + \frac{2}{3}x \right) = \frac{5}{3} \frac{dx}{dt} = \frac{5}{3}(5) = \frac{25}{3} \text{ft/s} \)

e) Given: at noon, ship A is 150 km west of ship B; ship A is sailing east at 35 km/h, ship B is sailing north at 25 km/h. \( t = \text{time} \) \( x = \text{distance} \) traveled by ship A, \( y = \text{distance} \) traveled by ship B, given that \( \frac{dx}{dt} = 35 \text{km/h} \) \( \frac{dy}{dt} = 25 \text{km/h} \).

b) the rate at which the distance between the ships is changing at 4:00 pm, \( z = \text{distance} \) between the ships, then we want to find \( \frac{dz}{dt} \) when \( t = 4 \text{h} \).

d) \( z^2 = (150-x)^2 + y^2 \Rightarrow 2z \frac{dz}{dt} = 2(150-x)(-\frac{dx}{dt}) + 2y \frac{dy}{dt} \)

e) at 4pm, \( x = 4 \times 35 = 140 \) and \( y = 4 \times 25 = 100 \Rightarrow z = \sqrt{101} 

So \( \frac{dz}{dt} = \frac{1}{z} \left[ (x-150) \frac{dx}{dt} + y \frac{dy}{dt} \right] = \frac{-10 \cdot 35 + 100 \cdot 25}{\sqrt{101}} = \frac{215}{\sqrt{101}} \approx 21.4 \text{km/hr} \)

30. Using \( O \) for the origin, we’re given \( \frac{dx}{dt} = -2 \text{ft/s} \), need to find \( \frac{dy}{dt} \) when \( x = -5 \). From Pyth. theorem, we have \[ \sqrt{x^2 + 12^2} + \sqrt{y^2 + 12^2} = 39, \] the total length of the rope.

differentiate w.r.t. \( t \), we get \[ \frac{x}{\sqrt{x^2 + 12^2}} \frac{dx}{dt} + \frac{y}{\sqrt{y^2 + 12^2}} \frac{dy}{dt} = 0, \] so
6b). Since \( f'(x) = 0 \) at \( x = 3 \) and \( f' \) changes from positive to negative there, \( f \) changes from increasing to decreasing and has a local maximum at \( x = 3 \). Since \( f'(x) = 0 \) at \( x = -1 \) and \( x = 4 \) and changes from negative to positive at both values, \( f \) changes from decreasing to increasing and has local minima at \( x = -1 \) and \( x = 4 \).

7. There is an inflection pt at \( x = 1 \) because \( f''(x) \) changes from negative to positive there, and one at \( x = 7 \) because \( f''(x) \) changes from pos. to neg. there.

8. a) \( f \) is increasing on the intervals where \( f'(x) > 0 \), namely \((2,4)\) and \((6,9)\).

b) \( f \) has a local maximum where it changes from increasing to decreasing, that is, where \( f' \) changes from pos. to neg. \((x=4)\). Also, where \( f' \) changes from neg. to pos., \( f \) has a local minimum \((x=2,6)\).

c) \( f' \) increasing \( \Rightarrow f'' \) positive \( \Rightarrow f \) is concave upward. This happens on \((1,3),(5,7),(8,9)\). Similarly, \( f \) is concave downward when \( f' \) decreasing, on \((0,1),(3,5),(7,8)\).

d) \( f \) has inflection pts at \( x = 1,3,5,7, \) and \( 8 \).

32. a) \( f'(x) = 3 - 3x^2 = -3(x^2 - 1) = -3(x+1)(x-1) \) \( f'(x) > 0 \) \( \Rightarrow \) \(-x < 1 \) and \( f'(x) < 0 \) \( \Rightarrow \) \( x < -1 \) or \( x > 1 \) f incr. on \((-1,1)\) and \( f \) decr. on \((-\infty,1) \cup (1,\infty)\).

b) \( f(-1) = 0 \) is a local min value and \( f(1) = 4 \) is a local max.

c) \( f''(x) = -6x \) \( \Rightarrow f''(x) > 0 \) on \((-\infty,0)\) and \( f''(x) < 0 \) on \((0,\infty)\). So, \( f \) is concave upward on \((-\infty,0)\) and concave downward on \((0,\infty)\). Inflection pt at \((0,2)\).

34. a) \( g'(x) = 24x^2 + 4x^3 = 4x^2(6+x) = 0 \) when \( x = -6,0 \). \( g'(x) > 0 \) \( \Rightarrow x > -6 \)

\((x \neq 0)\) and \( g'(x) < 0 \) when \( x < -6 \) \( g \) decr. \((-\infty,-6)\) and \( g \) incr. \((-6,\infty)\) with a horizontal tangent \( x = 0 \).

b) \( g(-6) = -232 \) is a local min. No local max. c) \( g''(x) = 48x + 12x^2 = 12x(4+x) = 0 \) when \( x = -4,0 \).

\( g \) is \( \cup (-\infty,-4) \cup (0,\infty) \). \( g \) is \( \cap (-4,0) \). Inflection pts at \((-4,-56)\) and \((0,200)\).
4.8 2) a) 

\[ C(x) = \frac{C(x)}{x} \rightarrow \text{can read } C(x) \text{ from graph.} \]

b) \( C(x) = C(x)/x \) is decreasing. min value of \( C(x) \) is at \( x = 7 \).
At \( x = 7 \), Marginal cost = Average cost.

1) a) Profit is maximized when \( R'(x) = C'(x) \).

b) \( P(x) = R(x) - C(x) \)

c) \( P'(x) \rightarrow \)

14. \( C(x) = 680 + 4x + 0.01x^2 \), \( P(x) = 12 - \frac{x}{500} \) \( \Rightarrow R(x) = 12x - \frac{x^2}{500} \).
Profit is maximized when \( R'(x) = C'(x) \)
\[ \Rightarrow 12 - \frac{x}{250} = 4 + 0.02x \Rightarrow x = \frac{1000}{3}. \]
\( P''(x) < 0 \) for \( P(x) \) to be maximum.
\( R''(x) = -\frac{1}{250}, \quad C''(x) = 0.02 \) \( \Rightarrow R''(x) > C''(x) \) which satisfies \( R''(x) - C''(x) < 0 \), \( x = \frac{1000}{3} \) is maximum.

16. \( C(x) = 10000 + 28x - 0.01x^2 - 0.026x^3, \quad P(x) = 90 - 0.02x \).
\( R(x) = 90x - 0.02x^2, \quad R'(x) = C'(x) \) \( \Rightarrow 90 - 0.04x = 28 - 0.02x + 0.06x \)

Solving for \( x \), \( x = 100 \),
Profit is maximized when \( P''(x) < 0 \),
\[ R''(x) - C''(x) = (90 - 0.04x) - (-0.02 + 0.012x) = 0.02 - 0.028x < 0 \] at this \( x \)
Hence Profit is maximized.
12. Distance traveled after 62 seconds.
\[ d = 185.10 + 319.5 + 447.5 + 742.12 + 1325.27 + 1445.3 \]
= 54,694 feet.

5.2.b) a) For right endpoints.
\[ \sum_{i=1}^{6} g(x_i) \cdot \Delta x = \left( g(-2) + g(-1) + g(0) + g(1) + g(2) + g(3) \right) \]
\[ \approx 1 - 0.5 - 1.5 - 1.5 - 0.5 - 2.5 - 0.5 \]
b) For left endpoints.
\[ g(-3) + g(-2) + g(-1) + g(0) + g(1) + g(2) \approx -1 \]
c) Midpoint:
\[ g(-2.5) + g(-1.5) + g(-0.5) + g(0.5) + g(1.5) + g(2.5) \approx -1.75 \]

10) \( \Delta x = \frac{\pi}{6} \Rightarrow \) end points 0, \( \frac{\pi}{6} \), \( \pi \).
\[ \int \sec^2(x/2) \, dx = \frac{\pi}{6} \left( \sec \frac{\pi}{3} + 3\pi/36 + \sec \frac{\pi}{2} + \sec \frac{\pi}{6} + \sec \frac{2\pi}{3} + \sec \frac{\pi}{6} \right) \]
\[ \approx 3.94. \]

30) a) \[ \int_{0}^{2} g(x) \, dx = \frac{1}{2} \cdot 4 + 2 = 4. \]
b) \[ \int_{0}^{2} g(x) \, dx = \frac{1}{2} \cdot \pi + 4 = -2\pi. \]
e) \[ \int_{0}^{\pi} g(x) \, dx = \frac{1}{2} \cdot 1.1 = \frac{1}{2}. \]

Final: \[ 4 - 2\pi + \frac{1}{2} = 4.5 - 2\pi. \]