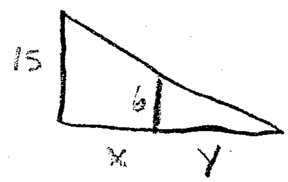
HW Oct 25 Math 120E Instructor: Grayson Fall 2001 (1)

§3.10 7. a) Given: a man 6 ft tall walks away from a street light mounted on a 15 ft tall pole at a rate of 5 ft/s. t = time x = distance from man to pole, $\frac{dx}{dt} = 5$ ft/s.

b) unknown: rate at which tip of shadow is moving when he is 40 ft from pole, y = distance from man to tip of his shadow, we want $\frac{d}{dt}(x+y)$ when $x = 40$ ft.

c)



d). By similar triangles, $\frac{15}{6} = \frac{x+y}{y} \Rightarrow$

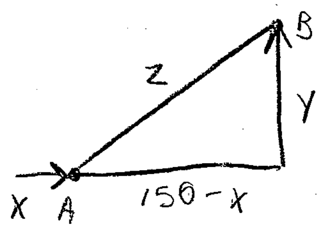
$15y = 6x + 6y \Rightarrow 9y = 6x \Rightarrow y = \frac{2}{3}x$

e). The tip of the shadow moves at a rate of $\frac{d}{dx}\left(x + \frac{2}{3}x\right) = \frac{5}{3}\frac{dx}{dt} =$

$\frac{5}{3}(5) = \frac{25}{3}$ ft/s

8 a) Given: at noon, ship A is 150 km west of ship B; ship A is sailing east at 35 km/h, ship B is sailing north at 25 km/h. t = time x = distance traveled by ship A, y = distance traveled by ship B, given that

$\frac{dx}{dt} = 35$ km/h $\quad \frac{dy}{dt} = 25$ km/h c).



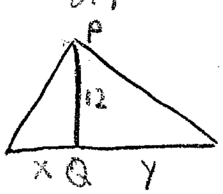
b) the rate at which the distance between the ships is changing at 4:00 pm. z = distance between the ships, then we want to find $\frac{dz}{dt}$ when $t = 4h$.

d). $z^2 = (150-x)^2 + y^2 \Rightarrow 2z\frac{dz}{dt} = 2(150-x)\left(-\frac{dx}{dt}\right) + 2y\frac{dy}{dt}$

e). at 4pm, $x = 4 \cdot 35 = 140$ and $y = 4(25) = 100 \Rightarrow z = \sqrt{10,100}$

So $\frac{dz}{dt} = \frac{1}{z}\left[(x-150)\frac{dx}{dt} + y\frac{dy}{dt}\right] = \frac{-10 \cdot 35 + 100 \cdot 25}{\sqrt{10,100}} = \frac{215}{\sqrt{101}} \approx 21.4$ km/hr

30.



Using Q for the origin, we're given $\frac{dx}{dt} = -2$ ft/s, need to find $\frac{dy}{dt}$ when $x = -5$. From Pyth. theorem, we have

$\sqrt{x^2+12^2} + \sqrt{y^2+12^2} = 39$, the total length of the rope.

Differentiate w.r.t t, we get $\frac{x}{\sqrt{x^2+12^2}}\frac{dx}{dt} + \frac{y}{\sqrt{y^2+12^2}}\frac{dy}{dt} = 0$, so

6b). Since $f'(x)=0$ at $x=3$ and f' changes from positive to negative there, f changes from increasing to decreasing and has a local maximum at $x=3$. Since $f'(x)=0$ at $x=^-1$ and $x=4$ and changes from negative to positive at both values, f changes from decreasing to increasing and has local minima at $x=^-1$ and $x=4$.

7. There is an inflection pt at $x=1$ because $f''(x)$ changes from negative to positive there, and one at $x=7$ because $f''(x)$ changes from pos. to neg. there.

8. a) f is increasing on the intervals where $f'(x)>0$, namely, $(2,4)$ and $(6,9)$.

b). f has a local maximum where it changes from increasing to decreasing, that is, where f' changes from pos. to neg. $(x=4)$. Also, where f' changes from neg. to pos., f has a local minimum. $(x=2$ and $x=6)$

c) f' increasing $\Rightarrow f''$ positive $\Rightarrow f$ is concave upward. This happens on $(1,3),(5,7)$, and $(8,9)$. Similarly, f is concave downward when f' decreasing, on $(0,1),(3,5)$, and $(7,8)$ d) f has inflection pts at $x=1,3,5,7$, and 8

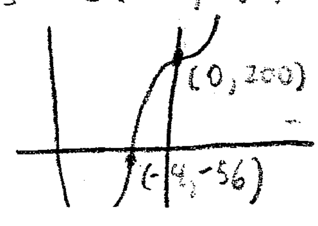
32. a) $f'(x)=3-3x^2=-3(x^2-1)=-3(x+1)(x-1)$ $f'(x)>0 \Rightarrow -1<x<1$ and $f'(x)<0$ $\Rightarrow x<-1$ or $x>1$ f incr. on $(-1,1)$ and f decr. on $(-\infty,1)\cup(1,\infty)$.

b) $f(-1)=0$ is a local min value and $f(1)=4$ is a local max.

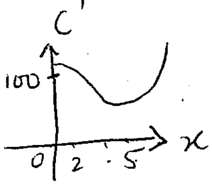
c) $f''(x)=^-6x \Rightarrow f''(x)>0$ on $(-\infty,0)$ and $f''(x)<0$ on $(0,\infty)$. So, f is concave upward on $(-\infty,0)$ and concave downward on $(0,\infty)$. Inflection pt at $(0,2)$

34. a) $g'(x)=24x^2+4x^3=4x^2(6+x)=0$ when $x=^-6,0$. $g'(x)>0 \Rightarrow x>^-6$ $(x\neq 0)$ and $g'(x)<0$ when $x<^-6 \Rightarrow g$ decr. $(-\infty,^-6)$ and g incr. $(-6,\infty)$ with a horizontal tangent $x=0$. b) $g(-6)=^-232$ is a local min No local max. c) $g''(x)=48x+12x^2=12x(x+4)=0$ when $x=^-4,0$.

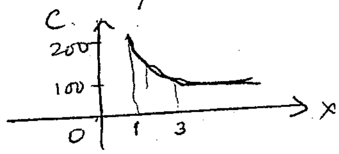
g is CU $(-\infty,-4)\cup(0,\infty)$. g is CD $(-4,0)$. Inflection pts. at $(-4,-56)$ and $(0,200)$. d)



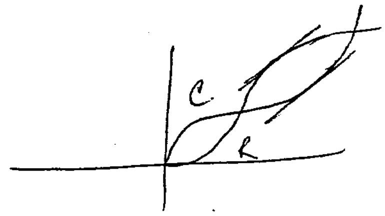
Homework Solutions (HW was Due 11/06/2001) - Math 120 - E - Fall 2001. (1)
Instructor: Prof. Grayson.

4.8 2). a) 

b). $c(x) = C(x)/x \rightarrow$ can read $C(x)$ from graph

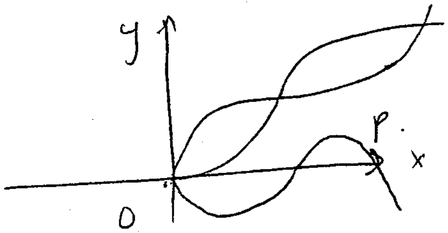


c). $c(x) = C(x)/x$ is decreasing. min value of $c(x)$ is at $x = 7$.
At $x = 7$, Marginal cost = Average cost.

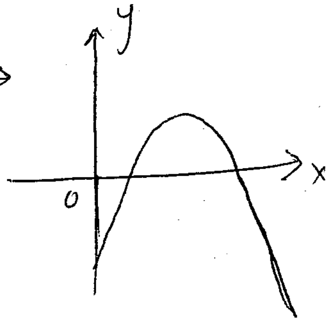


1) a) Profit is maximized when $R'(x) = C'(x)$.

b). $P(x) = R(x) - C(x)$



c). $P'(x) \rightarrow$



14). $C(x) = 680 + 4x + .01x^2$, $p(x) = 12 - x/500 \Rightarrow R(x) = 12x - x^2/500$.
Profit is maximized when $R'(x) = C'(x)$
$\Rightarrow 12 - x/250 = 4 + .02x \Rightarrow x = 1000/3$.

$P''(x) < 0$ for $P(x)$ to be maximum
$R''(x) = -1/250$, $C''(x) = .02$, $\Rightarrow R''(x) < C''(x)$ which.
Satisfies $R''(x) - C''(x) < 0$. $x = 1000/3$ is maximum.

16). $C(x) = 10000 + 28x - .01x^2 + .002x^3$, $p(x) = 90 - .02x$.
$R(x) = 90x - .02x^2$, $R'(x) = C'(x) \Rightarrow 90 - .04x = 28 - .02x + .006x$
Solving for x, $x = 100$,
Profit is maximized when $P''(x) < 0$,
$R''(x) - C''(x) = (\cancel{90} - .04x) - (-.02 + .012x) = .02 - .028x < 0$ at this x
Hence Profit is maximized.

12. Distance d after 62 seconds.

$d \simeq 185.10 + 319.5 + 447.5 + 742.12 + 1325.27 + 1445.3$

$= 54,694$ feet.

5.2 6) a). for right end points.

$$\sum_{i=1}^{6} g(x_i) \cdot \Delta x = 1\left(g(-2) + g(-1) + g(0) + g(1) + g(2) + g(3)\right)$$

$$\simeq 1 - 0.5 - 1.5 - 1.5 - 0.5 + 2.5 = 0.5$$

b) for left endpoints

$g(-3) + g(-2) + g(-1) + g(0) + g(1) + g(2) = -1$

c). Mid point:

$g(-2.5) + g(-1.5) + g(-.5) + g(.5) + g(1.5) + g(2.5) \simeq -1.75$

10) $\Delta x = \pi/6 \Rightarrow$ end points $0, \pi/6 \ldots \pi$.

$$\int_0^{\pi} \sec(x/3)\,dx \simeq \pi/6\left(\sec \pi/36 + 3\pi/36 + \sec 5\pi/36 + \sec 7\pi/36 + \sec 9\pi/36 + \sec 11\pi/36\right)$$

$\simeq 3.94$.

30) a). $\int_0^2 g(x)\,dx = \frac{1}{2} \cdot 4 \cdot 2 = 4$.

b) $\int_2^6 g(x)\,dx = \frac{1}{2} \cdot \pi\, 4 = -2\pi$.

e) $\int_6^{\cancel{7}} g(x)\,dx = \frac{1}{2} \cdot 1 \cdot 1 = \frac{1}{2}$

Final: $4 - 2\pi + \frac{1}{2} = 4.5 - 2\pi$.