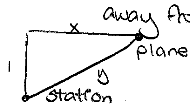
3.10

5. a) Given: altitude = 1 mile

speed of the plane = 500 mile per hour = dx/dt

b) unknown: rate at which the distance between the station and the plane is increasing when it's 2 miles away from the station (dy/dt)

c) 

d) By Pythagorean Theorem, $y^2 = x^2 + 1$

$\Rightarrow \frac{d}{dt}(y^2) = \frac{d}{dt}(x^2+1) \Rightarrow \frac{dy}{dt}\left[\frac{d}{dy}(y^2)\right] = \frac{dx}{dt}\left[\frac{d}{dx}(x^2+1)\right]$

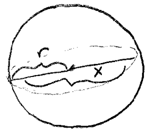
$\Rightarrow \frac{dy}{dt}(2y) = \frac{dx}{dt}(2x)$

e) $\frac{dy}{dt} = \frac{x}{y}\frac{dy}{dt} = 500\,\frac{x}{y}$ when $y = 2$, $x = \sqrt{(2)^2 - (1)^2} = \sqrt{3}$

$= 500\left(\frac{\sqrt{3}}{2}\right) = 250\sqrt{3} \approx 433 \text{ mi/h}$

6. a) Given: rate of change of surface area = $-1 \text{ cm}^2/\text{min}$ (dS/dt)

b) unknown: rate of change of diameter when diameter = 10 cm (dx/dt)

c) 

d) if r = radius, then $r = \frac{1}{2}(x)$

$S = 4\pi r^2 = 4\pi\left(\frac{x}{2}\right)^2 = 4\pi\left(\frac{x^2}{4}\right) = x^2\pi$

$\frac{dS}{dt} = 2x\pi\,\frac{dx}{dt}$

e) $\frac{dx}{dt} = \frac{dS}{dt}\cdot\frac{1}{2x\pi} = -1\cdot\frac{1}{2(10)\pi} = -\frac{1}{20\pi}$ cm/min

or rate of decrease $= +\frac{1}{20\pi}$ cm/min

3.11

6 $f(x) = \ln x$, $a = 1$ $f'(x) = 1/x$ $f(1) = 0$ $f'(1) = 1$

$L(x) = f(1) + f'(1)(x-1) = 0 + 1(x-1) = x-1$

8 $f(x) = \sqrt[3]{x} = x^{1/3}$, $a = -8$ $f'(x) = 1/3(x^{-2/3})$ $f(-8) = -2$ $f'(-8) = 1/12$

$L(x) = f(-8) + f'(-8)(x-(-8)) = -2 + 1/12(x+8) = x/12 - 4/3$

32 $y = f(x) = \sqrt[3]{x} + \sqrt[4]{x}$ $f'(x) = 1/3(x^{-2/3}) + 1/4(x^{-3/4})$

use linear approximation of f at 1

$L(x) = f(1) + f'(1)(x-1) = 2 + 7/12(x-1)$

$\sqrt[3]{1.02} + \sqrt[4]{1.02} \approx L(1.02) = 2 + 7/12(1.02 - 1) = 2.0117$

34 $f(x) = x^6$ $f'(x) = 6x^5$

use linear approximation of f at 2

$L(x) = f(2) + f'(2)(x-2) = 64 + 32(x-2)$

$(1.97)^6 \approx L(1.97) = 64 + 32(1.97 - 2) = 65.04$

36 $f(x) = \ln x$ $f'(x) = 1/x$

use linear approximation of f at 1

$L(x) = f(1) + f'(1)(x-1) = 0 + 1(x-1) = x-1$

$\ln 1.07 \approx L(1.07) = 1.07 - 1 = 0.07$

38 If $y = x^6$, then $y' = 6x^5$. The tangent line approximation at (1,1) has slope 6. If the change in x is 0.01, then change in y on the tangent line is 0.06. so approximating $(1.06)^6$ with 1.06 is reasonable

46 a) $f(x) = \sin x$ $f'(x) = \cos x$ $f(0) = 0$ $f'(0) = 1$

so $f(x) \approx f(0) + f'(0)(x-0) = 0 + 1(x-0) = x$

b) we want to know the value of x for which $y = x$ approximates $y = \sin x$ with less than a 2% difference.

In other words, we want values of x for which

$$\left|\frac{x - \sin x}{\sin x}\right| < 0.02 \longleftrightarrow \begin{cases} 0.98 \sin x < x < 1.02 \sin x & \text{, for } \sin x > 0 \\ 1.02 \sin x < x < 0.98 \sin x & \text{, for } \sin x < 0 \end{cases}$$

By graphing x, 1.02 sin x, and 0.98 sin x, estimate where x intersects the other 2 functions.

$y = x$ intersects $y = 1.02 \sin x$ at about $x = .344$

46 b) continued . . .

by symmetry $y=x$ intersects $y=0.98\sin x$ at about $x=-0.344$

$0.344\left(\frac{180°}{\pi}\right) \approx 19.7° \approx 20°$ so this verify the statement

4.1

4 absolute maximum : e absolute minimum : t

local maxima : c, e, s local minima : b, c, d, r

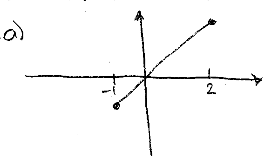
neither maxima nor minimum : a

6 absolute maximum value : $f(7)=5$ absolute minimum value : $f(1)=0$

local maximum values : , $f(0)=2$, $f(3)=4$, $f(5)=3$

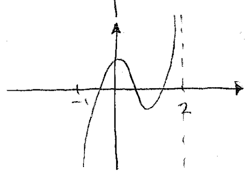
local minimum values : $f(1)=0$, $f(4)=2$, $f(6)=1$

12 a)

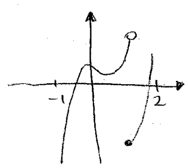


local maximum can't occur at an endpoint

b)

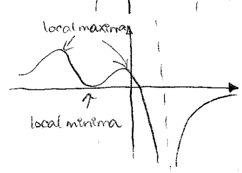


or

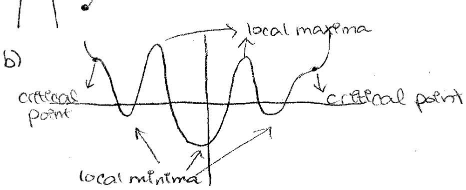


By extreme value theorem, f must not be continuous

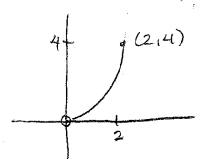
14 a)



b)



18



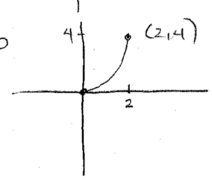
$f(x)=x^2 \quad 0<x\leq 2$

absolute maximum : $f(2)=4$

no local maximum or minimum or absolute minimum

$f(x)=x^2 \quad 0\leq x\leq 2$

20



absolute maximum : $f(2)=4$

absolute minimum : $f(0)=0$

no local maximum or minimum

32 $f(x) = 5 + 6x - 2x^3$ $\quad f'(x) = 6 - 6x^2 = 6(1-x)(1+x)$

critical numbers : $f'(x) = 0 = 6(1-x)(1+x)$ $\quad x = \pm 1$

34 $f(x) = 4x^3 - 9x^2 - 12x + 3$ $\quad f'(x) = 12x^2 - 18x - 12 = 6(2x^2 - 3x - 2) = 6(2x+1)(x-2)$

critical numbers : $f'(x) = 0 = 6(2x+1)(x-2)$ $\quad x = -\frac{1}{2}, 2$

50 $f(x) = x^3 - 3x + 1$, $[0,3]$ $\quad f'(x) = 3x^2 - 3$

$f'(x) = 0 = 3(x^2 - 1)$ $\quad x = \pm 1$, but -1 is not in $[0,3]$.

Test end points and critical points : $f(0) = 1$, $f(1) = -1$, $f(3) = 19$

so $f(3) = 19$ is absolute maximum and $f(1) = -1$ is absolute minimum

52 $f(x) = 18x + 15x^2 - 4x^3$, $[-3, 4]$ $\quad f'(x) = 18 + 30x - 12x^2 = 6(3-x)(1+2x)$

$f'(x) = 0 = 6(3-x)(1+2x)$ $\quad x = 3, -\frac{1}{2}$

Test end points and critical points : $f(-3) = 189$ $\; f(-\frac{1}{2}) = -\frac{19}{4}$, $f(3) = 81$

$f(4) = 56$ so $f(-3) = 189$ is the absolute maximum & $f(-\frac{1}{2}) = -\frac{19}{4}$

is the absolute minimum.

54 $f(x) = 3x^5 - 5x^3 - 1$, $[-2, 2]$ $\quad f'(x) = 15x^4 - 15x^2 = 15x^2(x+1)(x-1)$

$f'(x) = 0 = 15x^2(x+1)(x-1)$ $\quad x = 0, \pm 1$

Test end points and critical points ; $f(-2) = -57$, $f(-1) = 1$, $f(0) = -1$,

$f(1) = -3$, $f(2) = 55$ so $f(-2) = -57$ is absolute minimum and

$f(2) = 55$ is the absolute maximum