$\boxed{\text{HW } 10/09}$

Solutions

Section 3.5

38 $y' = (\sin(\underbrace{\sin(\sin x)}_{g(x)}))'$

$= \cos(\sin(\sin x)) \cdot (\sin(\underbrace{\sin x}_{g(x)}))'$

$= \cos(\sin(\sin x)) \cdot \cos(\sin x) \cdot (\sin x)'$

$= \cos(\sin(\sin x)) \cdot \cos(\sin x) \cdot \cos x$

Comments: There are so many composite functions there, if you are not sure the answer, why not do it step by step!

40 I will apply the comments above to this problem.

$y' = (\sqrt{\underbrace{x+\sqrt{x+\sqrt{x}}}_{g(x)}})'$

$= \frac{1}{2}(x+\sqrt{x+\sqrt{x}})^{-\frac{1}{2}}(x+\sqrt{x+\sqrt{x}})'$

$= \frac{1}{2}(x+\sqrt{x+\sqrt{x}})^{-\frac{1}{2}}(1+(\sqrt{\underbrace{x+\sqrt{x}}_{g(x)}})')$

$= \frac{1}{2}(x+\sqrt{x+\sqrt{x}})^{-\frac{1}{2}}(1+\frac{1}{2}(x+\sqrt{x})^{-\frac{1}{2}}(x+\sqrt{x})')$

$= \frac{1}{2}(x+\sqrt{x+\sqrt{x}})^{-\frac{1}{2}}(1+\frac{1}{2}(x+\sqrt{x})^{-\frac{1}{2}}(1+\frac{1}{2}x^{-\frac{1}{2}}))$

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$$y' = \left(2^{3^{x^2}}\right)' \qquad g(x) = 3^{x^2},\quad 2 \to \text{a number}$$

$$= \ln 2^{3} \cdot 2^{3^{x^2}} \cdot (x^2)'$$

$$= 3 \cdot \ln 2 \cdot 2^{3^{x^2}} \cdot 2x$$

$$= 6 \ln 2 \cdot x \cdot 2^{3^{x^2}}$$

58 (a)

$$h'(x) = f'(\underbrace{f(x)}_{g(x)}) \cdot (f(x))'$$

$$= f'(f(x)) \cdot f'(x)$$

so $h'(2) = f'(f(2)) \cdot f'(2)$

go to the graph, we found $f(2) = 1$ and $f'(2) = -1$.

so $h'(2) = f'(1) \cdot (-1)$

$$= (-1) \cdot (-1)$$

$$= 1$$

Comment: Since we can only estimate $f'(1)$ and $f'(2)$, once your estimation about them are not too far away -1, then your answer is also fine.

(b) $g'(x) = f'(x^2) \cdot 2x$

So $g'(2) = f'(4) \cdot 4 \; 4 \cdot \frac{3}{2} = 6$

Section 3.6

6 Differentiating on both sides with respect to x,

$$2x - 2y \cdot y' = 0$$

so

$$y' = \frac{x}{y}$$

10

$$5y^4 \cdot y' + 2x \cdot y^3 + x^2 \cdot 3y^2 \cdot y' = 2xye^{x^2} + e^{x^2} y'$$

$$5y^4 \cdot y' + 3x^2 y^2 \cdot y' - e^{x^2} y' = 2xye^{x^2} - 2xy^3$$

$$(5y^4 + 3x^2 y^2 - e^{x^2})\, y' = 2xye^{x^2} - 2xy^3$$

$$y' = \frac{2xye^{x^2} - 2xy^3}{5y^4 + 3x^2y^2 - e^{x^2}}$$

14 $$\frac{d}{dx}\left(\sqrt{1+x^2y^2}\right) = \frac{d}{dx}(2xy)$$

$$\frac{1}{2}(1+x^2y^2)^{-\frac{1}{2}}\cdot\frac{d}{dx}(1+x^2y^2) = 2y+2xy'$$

$$\frac{1}{2}(1+x^2y^2)^{-\frac{1}{2}}\cdot(2xy^2+x^2\cdot 2y\cdot y') = 2y+2xy'$$

$$xy^2\cdot(1+x^2y^2)^{-\frac{1}{2}} + x^2y(1+x^2y^2)^{-\frac{1}{2}}\cdot y' = 2y+2xy'$$

$$x^2y(1+x^2y^2)^{-\frac{1}{2}}\cdot y' - 2xy' = 2y - xy^2\cdot(1+x^2y^2)^{-\frac{1}{2}}$$

$$\left((x^2y(1+x^2y^2)^{-\frac{1}{2}}-2x\right)y' = 2y - xy^2\cdot(1+x^2y^2)^{-\frac{1}{2}}$$

$$y' = \frac{2y - xy^2\cdot(1+x^2y^2)^{-\frac{1}{2}}}{x^2y(1+x^2y^2)^{-\frac{1}{2}}-2x}$$

Now multiply $(1+x^2y^2)^{\frac{1}{2}}$ on both dinominator and numerator:

$$y' = \frac{2y(1+x^2y^2)^{\frac{1}{2}} - xy^2}{x^2y-(1+x^2y^2)^{\frac{1}{2}}\cdot 2x}$$

$$= \frac{y\left(2(1+x^2y^2)^{\frac{1}{2}} - xy\right)}{x\left(xy-(1+x^2y^2)^{\frac{1}{2}}\right)}$$

$$= -\frac{y}{x}$$

18 $$\frac{d}{dx}\left(x\cos y + y\cos x\right) = 0$$

$$\cos y + x\cdot(-\sin y)\cdot y' + y'\cos x + y\cdot(-\sin x) = 0$$

$$(\cos x - x\cdot\sin y)\, y' = y\cdot\sin x - \cos y$$

$$y' = \frac{y\sin x - \cos y}{\cos x - x\cdot\sin y}$$

28. Find y' first.

$$\frac{d}{dx}\left(x^{\frac{2}{3}} + y^{\frac{2}{3}}\right) = \frac{d}{dx}(4)$$

$$\frac{2}{3}x^{-\frac{1}{3}} + \frac{2}{3}y^{-\frac{1}{3}}\cdot y' = 0$$

so $$y' = -\frac{x^{-\frac{1}{3}}}{y^{-\frac{1}{3}}}$$

Thus $$y'(-3\sqrt{3}) = -\frac{(-3\sqrt{3})^{-\frac{1}{3}}}{1^{-\frac{1}{3}}} = \sqrt{3}^{-1} = \frac{1}{\sqrt{3}}$$

so the equation is just

$$y - 1 = \frac{1}{\sqrt{3}}(x + 3\sqrt{3})$$

30 $$\frac{d}{dx}\left(x^2y^2\right) = \frac{d}{dx}\left((y+1)^2(4-y^2)\right)$$

$$2xy^2 + x^2\cdot 2y\cdot y' = \frac{d}{dx}\left((y+1)^2\right)\cdot(4-y^2) + (y+1)^2\frac{d}{dx}(4-y^2)$$

$$2xy^2 + 2x^2y\cdot y' = 2(y+1)y'\cdot(4-y^2) - 2y(y+1)^2\cdot y'$$

$$2xy^2 + 2x^2y\cdot y' = -2(y+1)(3y^2+2y-4)\,y'$$

$$y' = \frac{-2xy^2}{2x^2y + 2(y+1)(3y^2+2y-4)}$$

So $\quad y'(0) = 0$

Thus the equation of the tangent line is

$$y = -2$$

38. Let's pick up one point (x_0, y_0) from the graph and find the tangent line.

$$\frac{d}{dx}\left(\sqrt{x}+\sqrt{y}\right)=\frac{d}{dx}\left(\sqrt{c}\right)$$

$$\frac{1}{2}x^{-\frac{1}{2}}+\frac{1}{2}y^{-\frac{1}{2}}y'=0$$

so

$$y'=-\sqrt{\frac{y}{x}}$$

Thus the slope at (x_0, y_0) is just $-\sqrt{\frac{y_0}{x_0}}$, so the equation of the tangent line is:

$$y-y_0=-\sqrt{\frac{y_0}{x_0}}\left(x-x_0\right)$$

We can find x-intercept when $y=0$,

$$0-y_0=-\sqrt{\frac{y_0}{x_0}}\left(x-x_0\right)$$

so

$$x=\sqrt{x_0 y_0}+x_0=\sqrt{x_0}\left(\sqrt{x_0}+\sqrt{y_0}\right)$$

Since (x_0, y_0) is on the graph, namely $\sqrt{x_0}+\sqrt{y_0}=\sqrt{c}$, x-intercept is $\sqrt{c}\sqrt{x_0}$.

Similarly, y-intercept is $\sqrt{c}\sqrt{y_0}$, so

$$\begin{aligned} x\text{-intercept} + y\text{-intercept} &= \sqrt{c}\sqrt{x_0}+\sqrt{c}\cdot\sqrt{y_0} \\ &= \sqrt{c}\left(\sqrt{x_0}+\sqrt{y_0}\right) \\ &= c \end{aligned}$$

Since (x_0, y_0) is arbitrary, the conclusion holds for any tangent line.

Section 3.7

2 $d = f$, $c = f'$, $b = f''$, $a = f'''$

The explanation is skipped.

4 d is the position function;
c is the velocity function;
b is the acceleration;
a is the jerk.

6. $f'(t) = 8t^7 - 42t^5 + 8t^3$

$f''(t) = 56t^6 - 210t^4 + 24t^2$

8 $y' = \sin\theta + \theta\cos\theta$

$y'' = \cos\theta + \cos\theta - \theta\sin\theta$

$= 2\cos\theta - \theta\sin\theta$

44 (a) $s = t^2 - t + 1 \Rightarrow v(t) = s'(t) = 2t - 1$

$\Rightarrow a(t) = v'(t) = 2$

(b) $a(1) = 2\ m/s^2$

(c) $v(t) = 2t - 1 = 0$ when $t = \frac{1}{2}$

and $a(\frac{1}{2}) = 2\ m/s^2$

50 (a) $x(t) = \frac{t}{1+t^2}$

$\Rightarrow v(t) = x'(t) = \frac{1-t^2}{(1+t^2)^2}$

$\Rightarrow a(t) = v'(t) = \frac{2t(t^2-3)}{(1+t^2)^3}$

$a(t) = 0 \Rightarrow 2t(t^2-3) = 0 \Rightarrow t = 0$ or $\sqrt{3}$

(c) The particle is speeding up when v and a have the same sign; that is, when $1 < t < \sqrt{3}$. The particle is slowing down when v and a have opposite signs; that is, when $0 < t < 1$ and when $t > \sqrt{3}$.

10 $f'(t) = \left(\frac{1+\ln t}{1-\ln t}\right)'$

$$= \frac{\frac{1}{t}(1-\ln t) - (1+\ln t)\cdot\left(-\frac{1}{t}\right)}{(1-\ln t)^2}$$

$$= \frac{(1-\ln t)+(1+\ln t)}{t\,(1-\ln t)^2}$$

$$= \frac{2}{t(1-\ln t)^2}$$

12 $h'(x) = \frac{1}{x+\sqrt{x^2-1}}\left(x+\sqrt{x^2-1}\right)'$

$$= \frac{1}{x+\sqrt{x^2-1}}\left(1+\frac{1}{2}(x^2-1)^{-\frac{1}{2}}\cdot 2x\right)$$

$$= \frac{1}{x+\sqrt{x^2-1}}\cdot\frac{\sqrt{x^2-1}+x}{\sqrt{x^2-1}}$$

$$= \frac{1}{\sqrt{x^2-1}}$$

14 $h'(y) = \frac{1}{y^3\sin y}\left(y^3\sin y\right)'$

$$= \frac{3y^2\sin y + y^3\cos y}{y^3\sin y}$$

$$= \frac{3\sin y + y\cos y}{y\sin y} \quad \left(\text{or } \frac{3}{y}+\cot y\right)$$

12

16 $y' = 2\ln\tan x \cdot (\ln\tan x)'$

$= \dfrac{2\ln\tan x}{\tan x} \cdot \sec^2 x$