Homework due OCT 4

CHAPTER 3.5 THE CHAIN RULE

(2) $y = \tan 3x$

$y = f(g(x))$ where $g(x) = 3x$, $f(u) = \tan u$

$g'(x) = 3$, $f'(u) = \sec^2 u$

$y' = f'(g(x)) \cdot g'(x) = \sec^2(3x) \cdot 3 = 3\sec^2(3x)$ □

(4) $y = \sqrt[3]{1+x^3} = (1+x^3)^{1/3}$

$y = f(g(x))$ where $g(x) = 1 + x^3$, $f(u) = u^{1/3}$

$g'(x) = 3x^2$, $f'(u) = \frac{1}{3} u^{-2/3}$

$y' = f'(g(x)) \cdot g'(x) = \frac{1}{\cancel{3}}(1+x^3)^{-2/3} \cdot \cancel{3}x^2 = \frac{x^2}{(1+x^3)^{2/3}}$ □

(6) $y = \sin(e^x) = f(g(x))$

$g(x) = e^x$, $g'(x) = e^x$

$f(u) = \sin u$, $f'(u) = \cos u$

$y' = f'(g(x)) \cdot g'(x) = \cos(e^x) \cdot e^x$ □

(8) $F(x) = (x^2 - x + 1)^3 = f(g(x))$

$g(x) = x^2 - x + 1$, $g'(x) = 2x - 1$

$f(u) = u^3$, $f'(u) = 3u^2$

$F'(x) = 3(x^2 - x + 1)^2 \cdot (2x - 1)$ □

(10) $f(t) = \dfrac{1}{(t^2-2t-5)^4} = (t^2-2t-5)^{-4}$

$f'(t) = -4\,(t^2-2t-5)^{-5} \cdot (2t-2) =$

$= 8(1-t)(t^2-2t-5)^{-5}$ $\square$

(12) $f(t) = \sqrt[3]{1+\tan t} = (1+\tan t)^{1/3}$

$f'(t) = \frac{1}{3}(1+\tan t)^{-\frac{2}{3}} \cdot \sec^2 t$ $\square$

(14) $y = a^3 + \cos^3 x$

We assume that a is a number, so a^3 is a number, too. Hence $(a^3)' = 0$.
And our function y is a function only of one variable x: $y = y(x) = a^3 + \cos^3 x$
Thus we are looking for derivative of y with respect to x

$y' = 0 + 3\cos^2 x \cdot (-\sin x) = -3\cos^2 x \sin x$ $\square$

(16) $y = 4\sec 5x$

$y' = 4(\sec 5x)' = 4 \cdot \sec 5x \cdot \tan 5x \cdot 5 =$

$= 20 \sec 5x \cdot \tan 5x$ $\square$

(18) $g(t) = (6t^2+5)^3 \cdot (t^3-7)^4$

Here we need to use product rule as well as chain rule.

$$g'(t) = \left[\frac{d}{dt}(6t^2+5)^3\right] \cdot (t^3-7)^4 + (6t^2+5)^3 \left[\frac{d}{dt}(t^3-7)^4\right] =$$

$$= \left[3(6t^2+5)^2 \cdot (12t)\right](t^3-7)^4 + (6t^2+5)^3 \left[4(t^3-7)^3 \cdot (3t^2)\right] =$$

$$= 36t\,(6t^2+5)^2(t^3-7)^4 + 12t^2(6t^2+5)^3(t^3-7)^3 =$$

$$= 12t\,(6t^2+5)^2(t^3-7)^3\left[3(t^3-7) + t(6t^2+5)\right] =$$

$$= 12t\,(6t^2+5)^2(t^3-7)^3\left[3t^3-21+6t^3+5t\right] =$$

$$= 12t\,(6t^2+5)^2(t^3-7)^3(9t^3+5t-21)$$

Factoring the polynomial is useful when looking for zeros of the derivative. $\square$

(20) $y = (x^2+1)\sqrt[3]{x^2+2} = (x^2+1)(x^2+2)^{1/3}$

$$y' = \left[(x^2+1)\right]' \cdot (x^2+2)^{1/3} + (x^2+1)\left[(x^2+2)^{1/3}\right]' =$$

$$= 2x \cdot (x^2+2)^{1/3} + (x^2+1) \cdot \frac{1}{3}(x^2+2)^{-2/3} \cdot 2x =$$

$$= 2x\,(x^2+2)^{1/3}\left(1 + (x^2+1)\frac{1}{3}(x^2+2)^{-1}\right) =$$

$$= 2x\,(x^2+2)^{1/3}\left(1 + \frac{x^2+1}{3(x^2+2)}\right) \quad \square$$

(22) $y = e^{-5x} \cdot \cos 3x$

$$y' = \frac{d}{dx}\left(e^{-5x}\right) \cos 3x + e^{-5x} \cdot \frac{d}{dx}\left(\cos 3x\right) =$$

$$= e^{-5x} \cdot (-5) \cdot \cos 3x + e^{-5x} \cdot (-\sin 3x) \cdot 3 =$$

$$= -5e^{-5x} \cdot \cos 3x - 3e^{-5x} \cdot \sin 3x =$$

$$= -e^{-5x}\left(5 \cos 3x + 3 \sin 3x\right) \qquad \square$$