Math 120-E – Homework due 09/27/2001

Solutions.

§ 3.1.

44. $f(x) = 2x^3 - 3x^2 - 6x + 7$

Has a horizontal tangent when $f'(x) = 0$.

$$f'(x) = 6x^2 - 6x - 6 = 0$$

$$\Rightarrow x^2 - x - 1 = 0$$

$$x = \frac{+1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}.$$

48. Find equations of both lines through $(2, -3)$ that are tangent to parabola $y = x^2 + x$

Solution:

$$y = x^2 + x$$

$$\Rightarrow y' = 2x + 1$$

Slope of tangent is $2a + 1$ at the point $(a, a^2 + a)$

Tangent passes through $(2, -3)$

$\Rightarrow$ slopes must be equal.

$$\Rightarrow 2a + 1 = \frac{a^2 + a + 3}{a - 2}.$$

$$\Rightarrow (2a+1)(a-2) = a^2 + a + 3.$$

$\Rightarrow \quad 2a^2 - 3a - 2 = a^2 + a + 3$

$\Rightarrow \quad a^2 - 4a - 5 = 0$

$a = -1, 5.$

a) If $a = -1$ point is $(-1, 0)$, slope $= -1$.
equation : $y - 0 = -1(x + 1)$

$\Rightarrow \underline{y = -x - 1.}$

b) If $a = +5$, point is $(5, 30)$, slope $= 11$.
equation : $y - 30 = 11(x - 5)$

$\Rightarrow \underline{y = 11x - 25.}$

61. $$\lim_{x \to 1} \frac{x^{1000} - 1}{x - 1}.$$

Soln :
Consider $$f'(1) = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1}$$

$$= \underbrace{\lim_{x \to 1} \frac{x^{1000} - 1}{x - 1}}_{\text{Given limit.}}$$

hence $f'(x) = 1000x^{999}$ (Power Rule).

$f'(1) = 1000 \cdot 1$

$= \boxed{1000}$

§ 3.2

4. $g(x) = \sqrt{x}\, e^x$

Soln:

$$g'(x) = \sqrt{x}\cdot e^x + e^x \cdot \frac{1}{2\sqrt{x}} = e^x\left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right).$$

6. $y = \dfrac{e^x}{1+x}$.

Soln:

$$y' = \frac{(1+x)\cdot e^x - e^x(1)}{(1+x)^2} = \frac{xe^x}{(1+x)^2}.$$

8. $f(u) = \dfrac{1-u^2}{1+u^2}$.

Soln:

$$f'(u) = \frac{(1+u^2)(-2u) - (1-u^2)(2u)}{(1+u^2)^2}.$$

$$= \frac{-2u - \cancel{2u^3} - 2u + \cancel{2u^3}}{(1+u^2)^2}$$

$$= \frac{-4u}{(1+u^2)^2}$$

10. $g(x) = (1+\sqrt{x})(x - x^3)$.

Soln:

$$g'(x) = \frac{1}{2\sqrt{x}}(x - x^3) + (1+\sqrt{x})(1-3x^2).$$

14. $y = \dfrac{4t+5}{2-3t}$

Soln:

$$y' = \frac{(2-3t)\,4 - (4t+5)(-3)}{(2-3t)^2}$$

$$= \frac{8 - \cancel{12t} + \cancel{12t} + 15}{(2-3t)^2} = \frac{23}{(2-3t)^2}.$$

18. $y = \dfrac{u^2-u-2}{u+1} = \dfrac{(u-2)\cancel{(u+1)}}{\cancel{(u+1)}} = u-2$

Soln:

$\underline{y' = 1}$

20. $y = \dfrac{e^x}{x+e^x}$

Soln.

$$y' = \frac{(x+e^x)\,e^x - e^x(1+e^x)}{(x+e^x)^2}$$

$$= \frac{e^x(x-1)}{(x+e^x)^2}$$

22. $f(x) = \dfrac{ax+b}{cx+d}$

Soln: $f'(x) = \dfrac{(cx+d)a - (ax+b)c}{(cx+d)^2}$

$= \dfrac{ad-bc}{(cx+d)^2}.$

24. $y = \dfrac{\sqrt{x}}{x+1}$ at $(4, 0.4)$

Soln.

$y' = \dfrac{(x+1)\cdot\frac{1}{2\sqrt{x}} - \sqrt{x}\cdot 1}{(x+1)^2} = \dfrac{\frac{1}{2\sqrt{x}} - \frac{\sqrt{x}}{2}}{(x+1)^2}.$

$y'\big|_{(4,0.4)} = \dfrac{\frac{1}{2\cdot\sqrt{4}} - \frac{\sqrt{4}}{2}}{25} = \dfrac{\frac{1}{4} - 1}{25} = \dfrac{-3}{100} = -.03$

Equation: $y - 0.4 = -.03(x-4).$

$\Rightarrow y = -.03x + 0.52.$

26. $y = e^x/x$ $(1, e).$

$y' = \dfrac{xe^x - e^x}{x^2}.$; $y'\big|_{(1,e)} = \dfrac{1.e^1 - e^1}{1^2} = 0.$

Equation: $y - e = 0(x-1)$

$\Rightarrow y = e.$

32. $f(3) = 4, \quad g(3) = 2, \quad f'(3) = -6, \quad g'(3) = 5$

Soln:

i) $(f+g)'(3) = f'(3) + g'(3) = \underline{\underline{-1}}.$

ii). $(fg)'(3) = f(3) \cdot g'(3) + f'(3)\, g(3)$

$= 4 \cdot 5 + (-6)(2)$

$= \underline{\underline{8}}.$

iii). $\left(f/g\right)'(3) = \dfrac{g(3) \cdot f'(3) - f(3)\, g'(3)}{g(3)^2}$

$= \dfrac{2(-6) - (4)(5)}{(2)^2} = \underline{\underline{-8}}.$

iv). $\left(\dfrac{f}{f-g}\right)'(3) = \dfrac{[f(3) - g(3)][f'(3)] - f(3)[f'(3) - g'(3)]}{[f(3) - g(3)]^2}.$

$= \dfrac{(4-2)(-6) - 4(-6-5)}{(4-2)^2}.$

$= \dfrac{-12 + 44}{4} = \underline{\underline{8}}$

36. Soln:

a). $y = x^2 f(x)$

$y' = x^2 f'(x) + 2x f(x).$

b). $y = \dfrac{f(x)}{x^2}$

$$y' = \frac{x^2 f'(x) - 2x \cdot f(x)}{(x^2)^2}.$$

c). $y = \dfrac{x^2}{f(x)}.$

$$y' = \frac{f(x)(2x) - x^2 (f'(x))}{[f(x)]^2}.$$

d). $y = \dfrac{1 + x f(x)}{\sqrt{x}}.$

$$y' = \frac{\sqrt{x}[1 + x f'(x) + f(x)] - [1 + x f(x)][1/2\sqrt{x}]}{x.}$$

$$= \frac{x^{1/2} + x^{3/2} f'(x) + x^{1/2} f(x) - \frac{1}{2}x^{-1/2} - \frac{x^{1/2} f(x)}{2}}{x.}$$

$$= \frac{x^{1/2} + x^{3/2} f'(x) + \frac{x^{1/2} f(x)}{2} - \frac{x^{-1/2}}{2}}{x.}$$

40. Find equations of tangent line to $y = \frac{x-1}{x+1}$ that are parallel to $x - 2y = 2$.

Soln:

$$y' = \frac{(x+1)\cdot 1 - (x-1)\cdot 1}{(x+1)^2}$$

$$= 2/(x+1)^2.$$

At $x = a$, $y = \frac{a-1}{a+1}$, slope of tangent $= \frac{2}{(a+1)^2}$

But tangent is parallel to

$$x - 2y = 2$$

$$\Rightarrow y = \frac{1}{2}x - 1.$$

$$\Rightarrow \frac{2}{(a+1)^2} = \frac{1}{2}.$$

$$\Rightarrow (a+1)^2 = 4 \Rightarrow a^2 + 2a - 3 = 0, \quad a = -3, +1$$

i) When $a = 1$, $y = 0$.

equation:

$$y - 0 = \tfrac{1}{2}(x-1).$$

$$y = \tfrac{1}{2}(x-1).$$

ii) When $a = -3$, $y = 2$.

equation of tangent $\Big\}$: $y - 2 = +\tfrac{1}{2}(x+3)$

$\Rightarrow y = +\frac{x}{2} + \frac{7}{2}$.