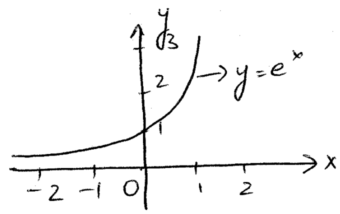
MATH 120 - E - HOMEWORK SOLUTIONS. DUE 09/25/2001

Section 3.1: Problems 2, 4, 8, 10, 18, 20, 24, 26 were graded.

2) a) Graph of e^x :



$f(0) = 1$.
$f'(0) = 1$.

b) $f(x) = e^x$ is an exponential function.
$g(x) = x^e$ is a power function

$f'(x) = e^x$; $g'(x) = ex^{e-1}$ (Power Rule).

4) a) 'e' is the number such that $\boxed{\lim_{h \to 0} \frac{e^h - 1}{h} = 1}$

b) $\lim_{h \to 0} \frac{2.7^h - 1}{h}$ and $\lim_{h \to 0} \frac{2.8^h - 1}{h}$.

Solutions are presented as:

x	$(2.7^x - 1)/x$
-.001	.9927
-.0001	.9932
.001	.9937
-.0001	.9933

x	$(2.8^x - 1)/x$
-.001	1.0291
-.0001	1.0294
.001	1.0301
.0001	1.0297

To 2 places of decimal, $\lim_{h \to 0} \frac{2.7^h - 1}{h} = .99$ and

$\lim_{h \to 0} \frac{2.8^h - 1}{h} = 1.03$. $\Rightarrow$ Both limits approach 1

as $.99 < 1 < 1.03$

and hence $\boxed{2.7 < e < 2.8}$ $\to$ Range for 'e'

Differentiate the following:

4) $F(x) = -4x^{10}$

$F'(x) = -4 \cdot 10\, x^{10-1} = \boxed{-40x^9}$ (Power Rule)

6) $g(x) = 5x^8 - 2x^5 + 6.$

$g'(x) = \underbrace{(5 \cdot 8 x^{8-1}) - (2 \cdot 5\, x^{5-1})}_{\text{using Power Rule}} + \underbrace{\frac{d}{dx}(6)}_{0}$ → since 6 is a constant

$= \boxed{40x^7 - 10x^4}$

8) $y = 5e^x + 3$

$y' = 5e^x.$

10) $R(t) = 5t^{-3/5}.$

$R'(t) = 5 \cdot \left(\frac{-3}{5}\right) \cdot t^{-3/5 - 1}$

$= -3t^{-8/5} = \boxed{\frac{-3}{t^{8/5}}}$

Note:
i) $x^{-m} = 1/x^m$
ii) Be careful with negative exponents!

12) $R(x) = \frac{\sqrt{10}}{x^7} = \sqrt{10}\,(x^{-7})$

$R'(x) = \sqrt{10}\,(-7)(x^{-7-1}) = -7\sqrt{10} \cdot x^{-8} = \boxed{\frac{-7\sqrt{10}}{x^8}}$

14) $y = \sqrt[3]{x} = x^{1/3}$

$y' = \frac{1}{3} x^{(1/3 - 1)} = \frac{1}{3} x^{-2/3} = \boxed{\frac{1}{3x^{2/3}}}$

16) $f(t) = \sqrt{t} - \frac{1}{\sqrt{t}} = t^{1/2} - t^{-1/2}$

$f'(t) = \frac{1}{2}\left(t^{1/2 - 1}\right) - \left(-\frac{1}{2}\right)\left(t^{-1/2 - 1}\right)$

$= \frac{1}{2}\left(t^{-1/2}\right) + \frac{1}{2}\left(t^{-3/2}\right)$

$= \frac{1}{2}\left[\frac{1}{\sqrt{t}} + \frac{1}{(\sqrt{t})^3}\right] = \boxed{\frac{1}{2\sqrt{t}}\left[1 + \frac{1}{t}\right]}$

18) $y = \frac{x^2 - 2\sqrt{x}}{x} = x - 2x^{-1/2}$ (Dividing each term of the numerator by 'x')

$y' = 1 - \cancel{2}\left(-\frac{1}{\cancel{2}}\right) \cdot \left(x^{-1/2 - 1}\right)$

$= 1 + x^{-3/2} = \boxed{1 + \frac{1}{x^{3/2}}}$

20) $y = \sqrt{x}(x - 1) = x^{3/2} - x^{1/2}$

$y' = \frac{3}{2}\left(x^{3/2 - 1}\right) - \frac{1}{2}\left(x^{1/2 - 1}\right)$

$= \frac{3}{2} x^{1/2} - \frac{1}{2} x^{-1/2} = \boxed{\frac{3}{2} x^{1/2} - \frac{1}{2x^{1/2}}}$

22) $y = x^{4/3} - x^{2/3}$

$y' = \frac{4}{3}(x^{4/3-1}) - \frac{2}{3}(x^{2/3-1})$

$= \frac{4}{3}x^{1/3} - \frac{2}{3}x^{-1/3}$

$= \boxed{\frac{2}{3}\left[2x^{1/3} - \frac{1}{x^{1/3}}\right]}$

24) $y = A + \frac{B}{x} + \frac{C}{x^2} = A + B(x^{-1}) + C(x^{-2})$

$y' = 0 + B(-1)(x^{-1-1}) + C(-2)(x^{-2-1})$

$\frac{d}{dx}(A) = 0$

$-Bx^{-2} - 2Cx^{-3} = \boxed{\frac{-B}{x^2} - \frac{2C}{x^3}}$

26) $u = \sqrt[3]{t^2} + 2\sqrt{t^3} = (t^2)^{1/3} + 2(t^3)^{1/2} = t^{2/3} + 2t^{3/2}$.

$u' = \frac{2}{3}(t^{2/3-1}) + 2 \cdot \frac{3}{2}(t^{3/2-1})$.

$= \frac{2}{3}t^{-1/3} + 3t^{1/2} = \boxed{\frac{2}{3t^{1/3}} + 3t^{1/2}}$

28) $y = e^{x+1} + 1 \quad = \quad \underbrace{e}_{\text{(a constant value)}} . e^{x} + 1$

$$y' = e . \frac{d}{dx}(e^{x}) + \underbrace{0}_{d/dx\,(1) = 0.}$$

$$= e . e^{x}$$

$$\boxed{= e^{x+1}}$$

General comments:

i) We always try to rewrite the given function like $y = \sqrt{x}(x+1)$ as $y = x^{1/2}(x+1) = x^{3/2} + x^{1/2}$. That's easier differentiation rather than using the Product Rule and making gory mistakes out of huge terms. Of course, for huge polynomials we turn to Product Rule for help!

ii) Derivative of any constant number is always zero, in case some of you are concerned.