MATH 120E, GRAYSON, EXAM #3, Nov 14, 2001

Name: ANSWERS Discussion section: ______

No calculators.
Simplification optional.
Useful formulas:

1	10	15	5	20	100
5	10	20	10	10	

$d/dx\,(\sin^{-1}(x)) = 1/\sqrt{1-x^2}$ $\quad d/dx\,(\sinh^{-1}(x)) = 1/\sqrt{1+x^2}$

$d/dx\,(\cos^{-1}(x)) = -1/\sqrt{1-x^2}$ $\quad d/dx\,(\cosh^{-1}(x)) = 1/\sqrt{x^2-1}$

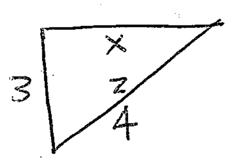
$d/dx\,(\tan^{-1}(x)) = 1/(1+x^2)$ $\quad d/dx\,(\tanh^{-1}(x)) = 1/(1-x^2)$

References to similar homework problems are provided.

1. (10 pts) Compute $\frac{d}{dx}\left(\sin^{-1}(2^{-x})\right)$.

$$\frac{1}{\sqrt{1-(2^{-x})^2}}(\ln 2)(2^{-x})(-1)$$

2. (15 pts; 3.10 #5) A plane flying horizontally at an altitude of 3 mi and a speed of 600 mi/h passes directly over Altgeld Hall. Find the rate at which the distance from the plane to the building is increasing when it is 4 mi away from the building.



$$z^2 = 9 + x^2$$

$$2z\frac{dz}{dt} = 2x\frac{dx}{dt}$$

$$\frac{dz}{dt} = \frac{x}{z}\frac{dy}{dt} = \frac{\sqrt{4^2-3^2}}{4}600 = 150\sqrt{7}\ \text{mi/h}$$

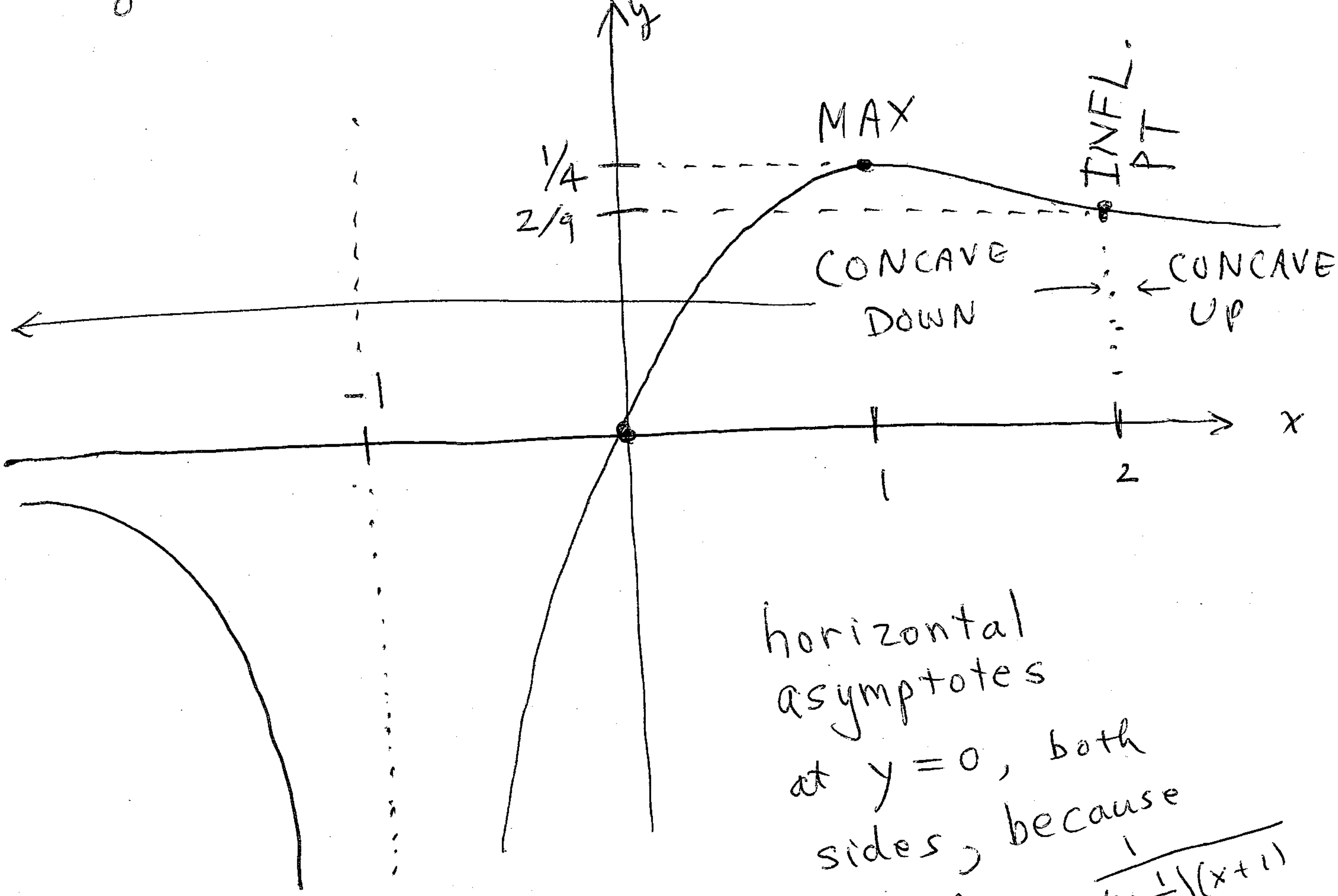
at that moment

3. (5 pts) A function f is given. You know only that $f(2) = 100$ and $f'(2) = -3$. What is your best guess for an approximate value of $f(2.01)$?

$$f(2.01) \approx f(2) + f'(2)\cdot(2.01-2) = 100 + (-3)(.01)$$
$$= 99.97$$

4. (20 pts; 4.5 #8) Sketch the curve $y = x/(x+1)^2$.
You may use these derivatives:
$y' = (1-x)/(x+1)^3$ and $y'' = 2(x-2)/(x+1)^4$.
Indicate, on your graph, concavity, any asymptotes, any local maximum or minimum values, and any inflection points.

x		-1		0		1		2	
y'	$- \; -$	$*$	$+ \; + \; +$		$+ \; + \; +$	0	$- \; - \; -$		$- \; -$
y''	$- \; -$	$*$	$- \; - \; -$		$- \; - \; -$		$- \; - \; -$	0	$+ \; +$



vertical asymptote

horizontal asymptotes at $y=0$, both sides, because

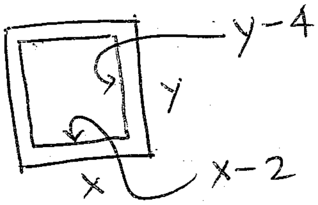
$$\lim_{x\to\pm\infty} \frac{x}{(x+1)^2} = \lim_{x\to\pm\infty} \frac{1}{\left(1+\frac{1}{x}\right)(x+1)} = 0$$

5. (10 pts ; 4.4 # 10) Find the limit.

$$\lim_{x \to 0} \frac{x + \tan(x)}{\sin(x)}$$

$$\overset{l'H}{=} \lim_{x \to 0} \frac{1 + \sec^2(x)}{\cos x} = \frac{1+1}{1} = 2$$

6. (20 pts ; 4.7 # 30) A poster is to have an area of 200 in² with 1-inch margins at the sides an with 2-inch margins at the top and bottom. What dimensions will give the largest printed area? Justify your answer by explaining why it's a global maximum.



$$xy = 200 \qquad y = 200/x$$

$$A = (x-2)(y-4) = (x-2)\left(200/x - 4\right)$$

$$= 200 - 4x - 400/x + 8$$

$$\frac{dA}{dx} = -4 + 400x^{-2} = 0$$

$$400x^{-2} = 4$$

$$100 = x^2$$

$x = 10$ critical point, <u>only one</u>

& $y = 200/10 = 20$

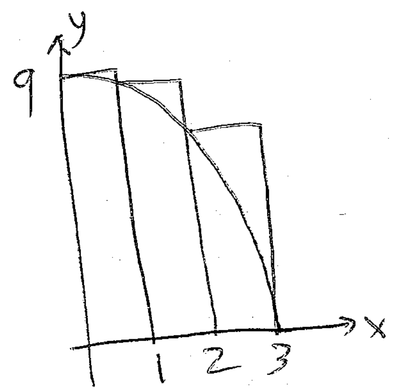
Justification:
<u>Closed interval test</u> : $2 \leq x \leq 50$

$A = 0$ at $x = 2$ & $x = 50$; (endpoints)

$A = (10-2)(20-4)$ at $x = 10$

$= 8 \cdot 16 = 128$ ↖ largest one

7. (10 pts ; 5.1 # 4) Estimate the area under the graph of $f(x) = 9 - x^2$ from $x = 0$ to $x = 3$ using three approximating rectangles and left endpoints. Sketch the graph and the rectangles. Is your estimate an underestimate or an overestimate?



$$9 \cdot 1 + (9 - 1^2) \cdot 1 + (9 - 2^2) \cdot 1$$

$$= 9 + 8 + 5 = 22$$

overestimate

8. (10 pts ; 5.3 # 22) Evaluate the integral.

$$\int_0^1 x^{4/5}\,dx$$

$$= \left.\frac{x^{9/5}}{9/5}\right]_0^1 = \frac{1^{9/5}}{9/5} = \frac{5}{9}$$