9.3/9.4 Main ideas

1. Circle the appropriate words when you have a choice:
   (a) You use a \( \sin / \cos \) Fourier series when you have an \textbf{even} function. The coefficients \( a_n/b_n \) are always 0.
   (b) You use a \( \sin / \cos \) Fourier series when you have an \textbf{odd} function. The coefficients \( a_n/b_n \) are always 0.

2. You can \underline{______________} and \underline{______________} a Fourier series term-by-term

3. To solve an endpoint boundary problem \( x'' + ax = f(t), x(0) = x(b) = 0 \)
   (a) Choose \( f(t) = \sum b_n \sin \left( \frac{n\pi t}{b} \right) \) or \( f(t) = \frac{a_0}{2} + \sum a_n \cos \left( \frac{n\pi t}{b} \right) \) (pick the one such that \underline{______________} satisfies \underline{______________})
   (b) Find the \underline{______________} above
   (c) Guess a Fourier series for \( x_p \) that \underline{______________}
   (d) Plug your guess for \( x_p \) into \( x'' + ax \) and equate \underline{______________}

4. Finding \( x_{sp} \) for the differential equation \( x'' + cx' + kx = F(t) \) is “the same” as always:
   (a) Find a \underline{______________} for \( F(t) \) (may attention to even and odd)
   (b) Guess \( x_{sp} = \frac{a_0}{2} + \sum a_n \cos \left( \frac{n\pi t}{L} \right) + b_n \sin \left( \frac{n\pi t}{L} \right) \)
   (c) Plug your guess for \( x_p \) into the differential equation and equate \underline{______________}
   (d) To find \( C \) and \( \alpha \), use the same process as in chapter 3 and deal with each pair \( a_n \cos \left( \frac{n\pi t}{L} \right) + b_n \sin \left( \frac{n\pi t}{L} \right) \) \underline{______________}

5. When does pure resonance occur? (This is bad for the same reason as before– bridges break)
Example

1. Find a Fourier series solution to $x'' - 3x = t, \ x'(0) = x'(3) = 0$
Practice Problems

1. Let \( f(x) = \begin{cases} 1/2 & 0 \leq x \leq 1 \\ 2x & 1 < x \leq 2 \end{cases} \)

(a) Assuming \( f \) above represents a full period, sketch the graph of the Fourier series expansion of \( f(x) \) over \([-2, 6]\) (Don’t compute this)

(b) Assuming \( f \) represents half the period of an odd function, sketch the graph of the Fourier sine series expansion of \( f(x) \) over \([2, -6]\) (Don’t compute this)

(c) Assuming \( f \) represents half the period of an even function, sketch the graph of the Fourier cosine series expansion of \( f(x) \) over \([2, -6]\)

(d) Compute the cosine series expansion of \( f(x) \)

2. Find a formal Fourier series solutions to

(a) \( x'' + 2x = t, x(0) = x(2) = 0 \)

(b) \( x'' + 2x = t, x'(0) = x' (\pi) = 0 \)

3. Find the steady periodic solution \( x_{sp}(t) \) of the differential equation \( x'' + 4x = F(t) \), where

(a) \( F(t) \) is the even function of period 4 such that \( F(t) = 2t \) if \( 0 < t < 2 \).

(b) \( F(t) \) is the odd function of period 4 such that \( F(t) = 2t \) if \( 0 < t < 2 \).

4. For the spring-mass system \( x'' + 4\pi^2 x = F(t) \), determine whether or not pure resonance can occur under the influence of the given external periodic force \( F(t) \). If it can occur, determine which coefficients need to be nonzero for resonance to occur.

(a) \( F(t) \) is an odd function of period 2 with \( F(t) = t \) for \( 0 < t < 1 \)

(b) \( F(t) \) is an odd function of period 2 with \( F(t) = 2t \) for \( 0 < t < 1 \)

(c) \( F(t) \) is the odd function of period 4 such that \( F(t) = 2t \) if \( 0 < t < 2 \).

(d) \( F(t) \) is an even function of period 2 with \( F(t) = t \) for \( 0 < t < 1 \)

5. Given a mass-dashpot-spring system with \( m = 3, c = 1, k = 30 \) and a odd forcing function of period 2 given \( F(t) = t - t^2 \) for \( 0 < t < 1 \), calculate the general solution (NOTE: This is still \( x_c + x_p \) including finding a formula for the coefficients and phase angles for the nonzero terms of \( x_{sp} \).

An Ending Thought: Whether you think you can or think you can’t, you’re right.
– Henry Ford