3.8 Main ideas

1. What is the difference between endpoint or boundary value problems and IVPs?

2. What is an eigenvalue in the boundary problem
   \[ y'' + p(x)y' + \lambda q(x)y = 0; \quad y(a) = 0, \quad y(b) = 0? \]

3. What is an eigenfunction associated to eigenvalue \( \lambda_0 \) in the boundary problem above?

Practice Problems

1. Show that there are no nontrivial solutions to the endpoint problem
   \[ y'' + 7y = 0; \quad y(0) = 0, \quad y(\pi) = 0. \]

2. Find a solution to the following problem. \( y'' + 9y = \cos x; \quad y'(0) = 5, \quad y(\pi/2) = -5/3. \) Is it unique?

3. Determine for the eigenvalues and eigenfunctions to the boundary value problem:
   \[ y'' + \lambda y = 0; \quad y(0) = 0, \quad y(L) = 0. \]
   (Hint: you should do this problem in three cases based on the value of \( \lambda \).)

4. Determine for the eigenvalues and eigenfunctions to the boundary value problem:
   \[ y'' + \lambda y = 0; \quad y'(0) = 0, \quad y'(L) = 0. \]
   (Hint: you should do this problem in three cases based on the value of \( \lambda \).)

5. Determine for the eigenvalues and eigenfunctions to the boundary value problem:
   \[ y'' + 2y' + \lambda y = 0; y(0) = y(1) = 0. \]
   Hint: The quadratic equation is your friend.

6. Consider the eigenvalue problem
   \[ y'' + \lambda y = 0; \quad y'(0) = 0, \quad y(1) + y'(1) = 0. \]
   All the eigenvalues are nonnegative, so write \( \lambda = \alpha^2 \) where \( \alpha \geq 0 \).
   (a) Show that \( \lambda = 0 \) is not an eigenvalue.
   (b) Show that \( y = A \cos \alpha x + B \sin \alpha x \) satisfies the endpoint conditions if and only if \( B = 0 \) and \( \alpha \) is a positive root for the equation \( \tan z = 1/z \).
Prep For Ch. 9

1. Practice the following integrals:

   (a) \[ \int \sin(nx) \, dx \]
   (b) \[ \int \cos(nx) \, dx \]
   (c) \[ \int x \sin(nx) \, dx \]
   (d) \[ \int x \cos(nx) \, dx \]

2. Given \( f(x) = \begin{cases} 6 & x > 1 \\ 3x^2 & x \leq 1 \end{cases} \) find \( \int_{-2}^{3} f(x) \, dx \).

Some Random Exam Practice

1. Find the general solution to \( y'' - 4y' - 12y = 3e^{5t} + \sin(2t) + te^{4t} \)

2. Find the general solution to \( 4y'' + 16y' + 17y = e^{-2t} \sin\left(\frac{t}{2}\right) + 6t \cos\left(\frac{t}{2}\right) \)

3. Find a general solution to \( 2y'' + 18y = 6 \tan(3t) \). What method is needed?

4. (On this problem, you can use a calculator to do some arithmetic. You will also need to watch your units). A 3 kg object (large chickens are apparently about 3 kg) is attached to a spring and will stretch the spring 392 mm by itself (with gravity). A dashpot will exert a force of 45 Newtons when the velocity is 50 cm/sec and an external force
   
   (a) Set-up the differential equation for this oscillator.
   
   (b) The object (chicken) is initially displaced 20 cm downward from its equilibrium position and given a velocity of 10 cm/sec upward. What are the initial conditions? (You should probably not try this at home)
   
   (c) Find the solution to this oscillator.
   
   (d) What is the long-term behavior (as \( t \to \infty \)) of this solution? Do the initial conditions matter?

5. Your exam will probably be 1 Fourier series (next week), 1 undetermined coefficients, 1 variation of parameters, 1 forced oscillator, 1 eigenvalue problem. Start thinking of questions on these now. Start seeking help for any of these topics if you need it now.

An Ending Thought: Success is not final, failure is not fatal: it is the courage to continue that counts.

– Winston Churchill