Main Ideas from § 3.1-3.3

I. Linear Independence of Functions

A. What is it? \( c_1 f_1 + c_2 f_2 + \ldots + c_n f_n = 0 \iff c_1 = c_2 = \ldots = c_n = 0 \)

B. How do you determine it?
   1. Check for a linear relation?
      \( \sin 2x, \sin x \cos x, xe^x - \ln x \)

C. The Wronskian
   Only works for \( n \) solutions to \( n \)th order homogeneous linear differential equations.
   \( \text{If identically zero, then linearly dependent.} \)
   \( \text{If nowhere vanishing, then linearly independent.} \)
   \[ W = \det \begin{pmatrix} f_1 & f_2 & \ldots & f_n \\ f_1' & f_2' & \ldots & f_n' \\ \vdots & \vdots & \ddots & \vdots \\ f_1^{(n-1)} & f_2^{(n-1)} & \ldots & f_n^{(n-1)} \end{pmatrix} \]

II. Why do we care?
   For the differential equation \( y^{(n)} + A_{n-1}(x)y^{(n-1)} + \ldots + A_1(x)y' + A_0(x)y = 0 \)
   If \( y_1, \ldots, y_n \) are \( n \) linearly independent solutions, the general solution is \( y = C_1 y_1 + C_2 y_2 + \ldots + C_n y_n \)

III. Homogeneous Equations with Constant Coefficients
   \( a_n y^{(n)} + a_{n-1} y^{(n-1)} + \ldots + a_1 y' + a_0 y = 0 \)

A. Guess \( y = e^{rx} \)

B. Plug in and get characteristic equation

C. Find the roots
   1. Rational root theorem: rational roots are factors of constant factors of leading term
   2. The quadratic formula: \( x^2 + x^3 + 1 \)
   3. Once you have a root, long divide

D. Use the roots to write the general solution.
   [Details on worksheet]