1. Solve the ODEs
   (a) $x^3 + 3y - xy' = 0$
   (b) $xy^2 + 3y^2 - x^2y' = 0$
   (c) $xy + y^2 - x^2y' = 0$
   (d) $y' = x^2 - 2xy + y^2$

2. Sketch a phase plane for the differential equation
   \[ \frac{dP}{dt} = -5P^2(P - 6)(P + 3)^3. \]

   Find the equilibrium solutions and discuss their stability.

3. Use the Wronskian to determine if $f(x) = 2\cos x + 3\sin x$ and $g(x) = 3\cos x - 2\sin x$ are linearly independent.

4. Find the general solution to
   (a) $y'' - 6y' + 13y = 0$
   (b) $4y'' - 12y' + 9y = 0$
   (c) $2y'' - 3y' = 0$
   (d) $y'' + y'' - y' - y = 0$

5. Find a linear homogeneous constant-coefficient ordinary (but super cool) differential equation with given general solution.
   (a) $y = Ae^{2x} + B\cos(2x) + C\sin(2x)$
   (b) $y = e^x((A + Bx)\cos 2x + (C + Dx)\sin 2x)$

6. Use variation of parameters to solve $y'' + 4y = \sin^2 x$. You can leave your answer in terms of two integrals.

7. Solve. You don’t need to find the coefficients for (c)
   (a) $y'' - 4y = \sinh x$
   (b) $y'' - 4y = \cosh 2x$
   (c) $y^{(4)} - 5y'' + 4y = e^x - xe^{2x}$
   (d) $y'' - y' - 6y = 2\sin 3x$

8. What differential equation models the motion of the following springs:
   (a) A spring with a mass of 2 kg attached and a spring constant of 3 with undamped and free motion.
   (b) A spring with a mass of 2 kg attached and a spring constant of 3 with a damping constant of 4 and free motion.
   (c) A spring with a mass of 2 kg attached and a spring constant of 3 with undamped with a force of $6\cos(2t)$ being applied.
(d) A spring with a mass of 2 kg attached and a spring constant of 3 with a damping constant of 4 with a force of $6 \cos(2t)$ being applied.

9. Fill out the following chart with the possible types of behavior. Some of the boxes have more than one case.

<table>
<thead>
<tr>
<th></th>
<th>Undamped</th>
<th>Damped</th>
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<tbody>
<tr>
<td>Free</td>
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<tr>
<td>Forced</td>
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10. A mass of 250 g is attached to the end of a spring that is stretched 25 cm by a force of 9N [Remember Hooke's law: $F = kx$]. At time $t = 0$, the body is pulled 1 m to the right [positive direction], stretching the spring, and set in motion with an initial velocity of $5m/s$ to the left. Find the equation for the motion of the spring, written as one oscillation [Make sure to watch your units]

11. A 2 kg mass is attached to a spring with spring constant 40 and a dashpot with damping constant 16. The mass is set into motion at initial position $5$ and initial velocity $4$. Find the equation of motion for the mass. Is it underdamped, overdamped, or critically damped?

12. Repeat the previous question with $m = 3$, $c = 30$, $k = 63$, $x_0 = 2$, $v_0 = 2$.

13. Find the general solution and state of resonance occurs.

(a) $x'' + 9x = 10 \cos 2t$  
(b) $x'' + 9x = 10 \cos 3t$

14. Find the transient and steady periodic solutions for the following systems. Which solution is which?

(a) $x'' + 4x' + 5x = 10 \cos 3t; x(0) = x'(0) = 0$  
(b) $x'' + 6x' + 13x = 10 \sin 5t; x(0) = x'(0) = 0$

15. Find the eigenvalue and eigenfunctions to the following problems.
(a) $y'' + 2y' + \lambda y = 0; y(0) = 0, y'(1) = 0$
(b) $y'' + 2y' + \lambda y = 0; y(0) = 0, y(1) = 0$

16. Find the Fourier series for

$$f(t) = \begin{cases} 
0, & -2 < t < 0 \\
t^2, & 0 < t < 2 
\end{cases}$$

17. Find formal Fourier series solutions of the endpoint value problems in

$$x'' + 2x = t, x(0) = x(2) = 0$$

18. Find formal Fourier series solutions of the endpoint value problems in

$$x'' + 2x = t, x'(0) = x'(2) = 0$$

19. Find $x_{sp}$ for $x'' + 2x = F(t)$, where $F(t)$ is the even function of period $2\pi$ such that $F(t) = \sin t$ if $0 < t < \pi$

20. Determine if resonance occurs in the system $x'' + 9x = F(t)$ where $F(t)$ is the odd function of period $2\pi$ with $F(t) = 1$ for $0 < t < \pi$.


**An Ending Thought:** *Do not worry about your difficulties with mathematics. I can assure you mine are still greater.*

– Albert Einstein