Hyperbolic Trig Functions
Why are they called that?

What are they and what do they look like?

Why do we care?
Some Trig Identities

- \( \sin(2u) = 2 \sin u \cos u \)
- \( \cos(2u) = \cos^2 u - \sin^2 u = 2 \cos^2 u - 1 = 1 - 2 \sin^2 u \)
- \( \sin^2 u = \frac{1}{2}(1 - \cos(2u)) \)
- \( \cos^2 u = \frac{1}{2}(1 + \cos(2u)) \)

Some Hyperbolic Trig Facts

- \( \cosh(0) = 1 \)
- \( \sinh(0) = 0 \)
- \( \cosh(-x) = \cosh(x) \)
- \( \sinh(-x) = -\sinh(x) \)
- \( \sinh(a) \cosh(b) - \cosh(a) \sinh(b) = \sinh(a - b) \)
- \( (A + B) \cosh(\alpha x) + (A - B) \sinh(\alpha x) = Ae^{\alpha x} + Be^{-\alpha x} \)
Practice Problems

Note: You should not need to calculate any integrals, but you might need some trig identities. You might also need to write down an integral.

1. Solve the differential equation below.

\[ u_{xx} + u_{yy} = 0 \quad 0 < x < 2\pi \quad 0 < y < \pi \]
\[ u_y(x,0) = u_y(x,\pi) = 0 \quad u(0,y) = 0 \quad u(2\pi,y) = \sin^2 y \]

**Hint:** For \( X \), use sinh and cosh because you are in a rectangle.

2. Solve the differential equation below.

\[ u_{xx} + u_{yy} = 0 \quad 0 < x < 10 \quad y > 0 \quad u(x,y) \text{ is bounded as } y \to \infty \]
\[ u_x(0,y) = u_x(10,y) = 0 \quad u(x,0) = 2 + 4 \cos \left( \frac{2\pi x}{5} \right) - \cos (\pi x) \]

**Hint:** For \( Y \), use \( A_n e^{\alpha_n y} + B_n e^{-\alpha_n y} \) because you are in a strip. What do you know about \( A_n \)?

3. Solve the differential equation below.

\[ u_{xx} + u_{yy} = 0 \quad 0 < x < 4 \quad 0 < y < 7 \]
\[ u(x,0) = u(x,7) = 0 \quad u(4,y) = 0 \quad u(0,y) = g(y) \]

4. The book likes to do everything in constants. Solve the following differential equation.

\[ u_{xx} + u_{yy} = 0 \quad 0 < x < a \quad 0 < y < b \]
\[ u_x(0,y) = u_x(a,y) = 0 \quad u_y(x,0) = 0 \quad u(x,b) = f(x) \]

5. Solve the differential equation below.

\[ u_{xx} + u_{yy} = 0 \quad 0 < x < a \quad y < 0 \quad u(x,y) \text{ is bounded as } y \to -\infty \]
\[ u(0,y) = u(a,y) = 0 \quad u(x,0) = f(x) \]

6. Work on the review problems.

**An Ending Thought:** Failure will never overtake me if my determination to succeed is strong enough.

- Og Mandino