1. Even and Odd Extensions: Find Fourier sine and cosine series of the following functions and sketch the graphs of the two extensions of $f$ to which these series converge. Note $f$ is defined on half the period of its extensions.

(a) $f(t) = 1 - t, \ 0 < t < 2$
(b) $f(t) = t^2, \ 0 < t < \pi$
2. Endpoint Value Problems: Find formal Fourier series solutions to the following

(a) $x'' + 4x = 1 - t, x'(0) = x'(2) = 0$

(b) $x'' - 3x = 1 - t, x(0) = x(2) = 0$
3. Finding $x_{sp}$

(a) Find $x_{sp}$ when $x'' + 3x = F(t)$ where $F(t)$ is the odd function of period $2\pi$ such that $F(t) = 2t$ if $0 < t < \pi$
(b) Find $x_{sp}$ when $x'' + 7x = F(t)$ where $F(t)$ is the even function of period 6 such that $F(t) = 4t$ if $0 < t < 3$
4. Resonance: Determine whether or not resonance occurs.

(a) \( x'' + 9\pi^2 x = F(t) \) where \( F(t) \) is the odd function of period 4 such that \( F(t) = 1 - t \) if \( 0 < t < 2 \).

(b) \( x'' + 9\pi^2 x = F(t) \) where \( F(t) \) is the even function of period 4 such that \( F(t) = 1 - t \) if \( 0 < t < 2 \).

(c) \( x'' + 9x = F(t) \) where \( F(t) \) is the even function of period \( 2\pi \) such that \( F(t) = t^2 \) if \( 0 < t < \pi \).

(d) \( x'' + 7x = F(t) \) where \( F(t) \) is the even function of period \( 2\pi \) such that \( F(t) = t^2 \) if \( 0 < t < \pi \).
5. By inspection: Find values for $c_0$ and $c_n$ to make each of the following equations true or explain why it isn’t possible (without calculating integrals).

(a) $2 \cos(5x) - \cos(3x) = c_0 + \sum_n c_n \cos \left( \frac{nx}{3} \right)$

(b) $4 \sin(x) + 7 \sin(5x) = \sum_n c_n \sin (nx)$

(c) $4 + 2 \cos \left( \frac{x}{2} \right) - \cos(3x) = 2c_0 + \sum_n \frac{c_n n}{5} \cos \left( \frac{nx}{5} \right)$

(d) $2 \sin \left( \frac{x}{2} \right) + 4 \cos(3x) = \sum_n \frac{c_n n^2}{16} \sin \left( \frac{nx}{4} \right)$
6. Additional Practice

(a) Let \( f(x) = \begin{cases} 1/2 & 0 \leq x \leq 1 \\ 2x & 1 < x \leq 2 \end{cases} \)

i. Assuming \( f \) above represents a full period, sketch the graph of the Fourier series expansion of \( f(x) \) over \([-2, 6]\) (Don’t compute this)

ii. Assuming \( f \) represents half the period of an odd function, sketch the graph of the Fourier sine series expansion of \( f(x) \) over \([2, -6]\) (Don’t compute this)

iii. Assuming \( f \) represents half the period of an even function, sketch the graph of the Fourier cosine series expansion of \( f(x) \) over \([2, -6]\)

iv. Compute the cosine series expansion of \( f(x) \)

(b) Find a formal Fourier series solutions to

i. \( x'' + 2x = t, \ x(0) = x(2) = 0 \)

ii. \( x'' + 2x = t, \ x'(0) = x'(-\pi) = 0 \)

(c) Find the steady periodic solution \( x_{sp}(t) \) of the differential equation \( x'' + 4x = F(t) \), where

i. \( F(t) \) is the even function of period 4 such that \( F(t) = 2t \) if \( 0 < t < 2 \).

ii. \( F(t) \) is the odd function of period 4 such that \( F(t) = 2t \) if \( 0 < t < 2 \).

(d) For the spring-mass system \( x'' + 4\pi^2 x = F(t) \), determine whether or not pure resonance can occur under the influence of the given external periodic force \( F(t) \). If it can occur, determine which coefficients need to be nonzero for resonance to occur.

i. \( F(t) \) is an odd function of period 2 with \( F(t) = t \) for \( 0 < t < 1 \)

ii. \( F(t) \) is an odd function of period 2 with \( F(t) = 2t \) for \( 0 < t < 1 \)

iii. \( F(t) \) is the odd function of period 4 such that \( F(t) = 2t \) if \( 0 < t < 2 \).

iv. \( F(t) \) is an even function of period 2 with \( F(t) = t \) for \( 0 < t < 1 \)

An Ending Thought: *Even if you are on the right track, you’ll get run over if you just sit there!*  

- Will Rogers