1. If $f$ is a periodic function of period $2L$, state the formulas for $a_0, a_n,$ and $b_n$ for the Fourier series of $f$. Once you have these, what is the Fourier series of $f$?

2. Don’t work harder than you need to. Calculate Fourier series for the following functions. This should take 30 seconds, literally.
   (a) $f(t) = 4$
   (b) $f(t) = \cos(4t)$
   (c) $f(t) = 3\sin(2t)$

3. Cara’s steps for finding a Fourier series (already in order)
   (a) Draw the graph of $f$ on the period given. Extend the graph at least some.
   (b) Use the graph to determine if $f$ is even or odd.
   (c) Use necessary integral formulas to calculate $a_0, a_n,$ and $b_n$.
   (d) Plug into formula for the Fourier series.

4. Suppose $f(t)$ is a period $2\pi$ function with $f(t) = \frac{\pi - t}{2}$ for $0 < t < 2\pi$.
   (a) Sketch the graph of $f(t)$ from $-2\pi$ to $2\pi$
   (b) Using the graph, is $f$ even, odd, or neither? What does this mean about $a_0, a_n,$ and $b_n$?
   (c) Calculate the Fourier series of $f(t)$.

5. Note for any $c$, $a_n = \frac{1}{L} \int_c^{c+2L} f(t) \cos \left( \frac{n\pi t}{L} \right) dt$ (and similarly for the other formulas). Thinking about definite integrals as area, why does this make sense? This means you can take whatever interval is most convenient but even/odd tricks only works for $[-L, L]$.

6. Suppose $f$ is a function with period $2\pi$ with $f(t) = t^2$ for $0 < t < 2\pi$.
   (a) Sketch the graph of $f$ from $-2\pi$ to $2\pi$.
   (b) From the graph, is $f$ even, odd, or neither?
   (c) Calculate the Fourier series of $f$.
   (d) Plug in 0 and $\pi$ to both sides to get formulas from $\sum_{n=1}^{\infty} \frac{1}{n^2}$ and $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$. Note for $t = 0$, the Fourier series converges to the “average” value at 0 which you can read off your graph.
   (e) Add and subtract your two expressions and then divide by 2 above to get find $\sum_{n\text{ odd}} \frac{1}{n^2}$ and $\sum_{n\text{ even}} \frac{1}{n^2}$. 

\begin{align*}
\sum_{n=1}^{\infty} \frac{1}{n^2} \quad \text{and} \quad \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}.
\end{align*}
7. Suppose $f$ is a period 8 function with $f(t) = 3 + t$ for $-4 < t < 4$. Calculate the Fourier series of $f$. (You might want to break $f$ into pieces and then add the pieces back at the end).

8. Find the form of a particular solution to $y'' - 2y' + 2y = e^x \sin(2x) + 2x + xe^x \sin(x)$. If you value your sanity, don’t attempt to find the coefficients.

9. Find all eigenvalues and the corresponding eigenfunctions for the boundary value problem
   \[ y'' + \lambda y = 0 \quad y(0) = 0 \quad y(2) + y'(2) = 0 \]

10. Calculate the Fourier series expansion for the function given below in a full period.
    \[
    f(t) = \begin{cases} 
    -t & -1 < t < 0 \\
    t & 0 < t < 1 
    \end{cases}
    \]

11. Find the general solution for $y'' + 4y = 3 + \cos(2x)$

12. Solve the IVP
    \[
    y'' + y' - 2y = 2x \quad y(0) = \frac{1}{2} \quad y'(0) = 2
    \]

13. Determine if the oscillator described by the following equation for $x(t)$ is overdamped, underdamped, or critically damped. Then draw a few possible solution curves.
    \[ x'' + 2x' + x = 0 \]

14. Find the general solution for $y''' + 4y' = 24x^2$

15. Find a general solution $x(t)$ for the following oscillator equation and then draw a possible solution curve. How would you change the equation so the oscillator is not in pure resonance?
    \[ x'' + 9x = 3 \cos(3t) \]

16. Find all eigenvalues and the corresponding eigenfunctions for the boundary value problem
    \[ y'' - 4y' + \lambda y = 0 \quad y'(0) = 0 \quad y(2) = 0 \]

17. Using variation of parameters, solve $y'' - 4y' + 4y = 2e^{2x}$

**An Ending Thought:** *He who doesn’t hope to win has already lost.*

– Simon Bolivar