1. Consider $y'' + 2y = 4e^{3x}$

   (a) Why do we call this equation nonhomogeneous?
   (b) What is the associated homogeneous equation?

2. Put the steps to solving a nonhomogeneous constant coefficient equation in order.

   ___ Add $y_c$ and $y_p$
   ___ Group like terms and equate coefficients on both sides
   ___ Find the general solution to the associated homogeneous equation. Call it $y_c$
   ___ Guess $y_p$ based on the RHS using undetermined coefficients
   ___ Use any initial conditions
   ___ Take derivatives and plug the guess into the differential equation.
   ___ Adjust the guess for $y_p$ based on $y_c$
   ___ Solve for the undetermined coefficients

3. If each of the following appears on the RHS, what do you guess in $y_p$

   (a) 4  
   (b) $x$  
   (c) $x^2$  
   (d) $e^{4x}$  
   (e) $\cos(3x)$  
   (f) $xe^{-x}$  
   (g) $\cos(2x) + \sin(7x)$  
   (h) $x\cos(2x) + \sin(2x)$

4. In the following, $y_c$ and the RHS of a nonhomogeneous equation are given. Make a guess for $y_p$. You do not need to solve for the coefficients.

   (a) $y_c = c_1 + c_2x + c_3e^{4x}$, RHS: $3x - e^{4x} + 6\sin(3x)$
   (b) $y_c = c_1\cos x + c_2\sin x$, RHS: $x\cos x$
   (c) $y_c = c_1e^{2x}\cos 3x + c_2e^{2x}\sin 3x$, RHS: $e^{2x} - \sin 3x$

5. Find the general solution to the following

   (a) $y'' + 2y = 4e^{3x}$
   (b) $y'' - 3y' + 2y = x^2 + x - 5$
   (c) $2y'' - y' + y = 3\sin(2x)$
   (d) $y'' + y = \sin x + x\cos x$
6. Set up an appropriate form of a particular solution $y_p$ but do not determine the value of the coefficients. **Note:** This requires finding $y_c$. Why?

(a) $y'' + 9y = \cos(3x)$
(b) $y'' + 6y' + 16y = e^{-3x} \sin(\sqrt{7}x)$
(c) $x'' - 2x' + 2x = e^{-t} \sin(2t) + 2t + te^{-t} \sin(t)$
(d) $y'' - 6y + 13y = xe^{3x} \sin(2x)$

7. In the following, $y_c$ and the RHS are given. Use $y_c$ to figure out the left hand side of the equation and then solve.

$$y_c = c_1 \cos x + c_2 \sin x,$$
$$\text{RHS: } \sin x + x \cos x$$

8. Find the general solution of the following using the method of variation of parameters. Why do you need to use this method?

(a) $y'' + y = \sec x$
(b) $y'' + y = \csc^2 x$

9. Bob student is asked to find the general solution to $y^{(3)} - y = e^x + 7$. For his particular solution he guess $y_p = x(Ae^x + B)$. What mistake, if any, has he made?

10. Sections 3.4 and 3.6 are basically sections 3.3 and 3.5 with more vocabulary.

(a) Springs are modeled by $mx'' + cx' + kx = F(t)$. What do $m, c, k$ and $F$ represent?
(b) When (in the differential equation) does a spring have free motion? What about forced motion?
(c) What kind of system do we call $c = 0$? What about $c \neq 0$?

11. We can tell a lot about the behavior just from examining these constants. For each of the following scenarios, think of a differential equation that models it and determine the behavior. Speculate about the general case.

(a) Free, undamped motion
(b) Free, damped motion (what are the three types of behavior)
(c) Forced, undamped motion (what are the two types of behavior)
(d) Forced, damped motion
12. A 1-kilogram mass is attached to a spring. The spring is 0.5m in length, and exerts a force of 8 N when stretched to 1m in length. The entire system is submerged in a liquid that imparts a damping force numerically equal to 10 times the instantaneous velocity.

(a) Find a differential equation which describes the motion of the mass. (Recall that Newton’s law says that $F = ma$ and Hooke’s law says $F = kx$)

(b) Determine the equations of motions if:
   i. the weight is released from rest 1 meter below the equilibrium position, and
   ii. the weight is released 1 meter below the equilibrium position with an upward velocity of 12 m/s.

13. Let $x'' + 25x = f(t); x(0) = 0, x'(0) = 100$ be the equations for a mass-spring system.

   (a) Let $f(t) = 0$. What is the period and frequency of the solution?
   (b) Let $f(t) \neq 0$. What is the natural frequency of this system?
   (c) Let $f(t) = 90 \cos 4t$. Solve the IVP and write your solution as the sum of two oscillations. Explain the long run behavior of this solution physically.
   (d) Let $f(t) = 90 \cos 5t$. Find the solution to this initial value problem. Explain the long run behavior of this solution physically. What phenomenon arises in this system?

14. Let $x'' + 8x' + 25x = 200 \cos t + 520 \sin t; x(0) = -30, x'(0) = -10$ be the equations for a mass-spring-dashpot system.

   (a) In contrast to the previous problem, what makes this a mass-spring-dashpot system? Explain physically the nature of the new term.
   (b) Find the general solution to the complementary problem.
   (c) Find the steady periodic solution $x_{sp}(t)$ and write $x_{sp}$ as a single sinusoidal function of the form $C \cos (\omega t - \alpha)$. Be sure to clearly define $\alpha$.
   (d) Find the transient solution $x_{tr}$ for this problem.

15. Let $x'' + 10x' + 650x = 100 \cos (\omega t)$ be the equations for a mass-spring-dashpot system.

   (a) Find the solution to the complementary problem.
   (b) Find the steady periodic solution $x_{sp}(t)$ and write $x_{sp}$ as a single sinusoidal function of the form $C \cos (\omega t - \alpha)$. Be sure to clearly define $\alpha$.
   (c) Using the amplitude function $C(\omega)$ of the steady periodic solutions, determine if practical resonance occurs for this system. If so, determine the frequency $\omega$ and the maximum amplitude.

**An Ending Thought:** *The surest way not to fail is to determine to succeed.*

– Richard Brinsley Sheridan