1. These again Solve the following equations. Ones with (⋆) have tricky integrals in their solutions but you should be able to start them. Implicit solutions are ok on ones with (♣).

(a) \(xy' = y(\ln x - \ln y)\)  
(b) (⋆) \((e^x + e^{-x})y' = y^2\)  
(c) (♣) \(y' = \sqrt{x + 2y}\)  
(d) (♣) \(y^3y' = (y^4 + 1)\cos x\)  
(e) (♣) \(y' = \frac{y - x}{y + x}\)  
(f) (♣) \(yy'' + (y')^2 = yy'\)  
(g) \(3y + x^4y' = 2xy\)  
(h) \(y' = x^2 - 2xy + y^2\)  
(i) \(y' = 3(y + 7)x^2\)  
(j) \(2y''' - y'' - 5y' - 2y = 0\) (hint: -1)  
(k) \(2y + (x + 1)y' = 3x + 3\)  
(l) \(y^{(4)} = 16y\)

2. A population \(P(t)\) of chinchillas has a birth rate of \(\beta = 0.001P\) (births per month per chinchilla) and a constant death rate of \(\delta\). If \(P(0) = 10,000\) and \(P'(0) = 8\),

(a) What are the equilibrium solutions of this model? Sketch a slope field or several solutions and indicate their stability.
(b) What will the long-term behavior of the population be?
(c) How long (in months) will it take this population to double to 20,000 chinchillas?

3. Let \(g(x) := \begin{cases} 3x, & 1 \leq x \leq 2; \\ 0, & 2 < x \leq 3 \end{cases}\). On the interval \([1, 3]\), find a continuous solution to the initial value problem

\[
\frac{dy}{dx} + \frac{1}{x}y = g(x); \\
y(1) = 1.
\]

4. For each of the following, clearly explain what each term means and briefly explain how they are different.

(a) an initial value problem vs. an ordinary differential equation
(b) a linear vs. a non-linear ODE
(c) a homogeneous ODE vs. a non-homogeneous ODE

5. Find an ordinary differential equation with the general solution \(y(x) = (A + Bx + Cx^2)e^{2x}\)

6. The roots of the characteristic equation to a linear ODE with constant coefficients are 

\(-2, 1, 1, 1, 2 \pm \sqrt{3}i, 2 \pm \sqrt{3}i\).
(a) What order was the original ODE?
(b) What is the general solution of the ODE?

7. Solve the initial value problem

\[ y'' - 2y' + y = 0 \]
\[ y(0) = 0 \]
\[ y'(0) = 2 \]

8. Find the solution curves for ordinary differential equation (ODE)

\[ \frac{dy}{dx} - x^2y^3 = 0. \]

9. Consider the initial value problem (IVP)

\[ ty' = -4y + 6t^2, \quad y(-1) = 3. \]

(a) Explain why this ODE is 1st-order linear.
(b) Solve the IVP.

10. Pick problems from your least favorite sections and practice them.

An Ending Thought: Magic is believing in yourself. If you can do that, you can make anything happen.

- Johann Wolfgang Von Goethe