1. Put the steps for solving any substitution problem in order.
   __ Solve the new problem
   __ Solve for an explicit solution if possible
   __ Determine the type of problem
   __ Substitute back into the original variables
   __ Make the appropriate substitution

2. Homogeneous equations
   (a) Which of the following criteria makes an equation homogeneous? Circle the two answers that apply.
      i. Every term has the same degree
      ii. $y$ only appears as $y^1$
      iii. You have a $y^n$ term
      iv. $y'$ can be written as a function of $\frac{y}{x}$
      v. You can factor out the $x$’s and the $y$’s
   (b) Which of the following are homogeneous?
      i. $xyy' = x^2 + 3y^2$
      ii. $x^2y' = x + y$
      iii. $(x + y)y' = 1$
      iv. $xy' = y + \sqrt{x^2 + y^2}$
   (c) What substitutions do you make for $y$? For $y'$? What kind of equation do you get?
   (d) Solve the above equations that are homogeneous.

3. The magic substitution: use as a last resort or if $y$ is inside another function and the equation is not homogeneous. For each differential equation below, guess a magic substitution and solve
   (a) $xe^yy' = 2(e^y + x^3e^{2x})$
   (b) $(2x \sin y \cos y)y' = 4x^2 + \sin^2 y$
   (c) $y' - (4x - y + 1)^2 = 0$
4. Reducible Second-Order Equations

(a) There are two types
   
   i. missing $y$: $0 = F(x, y', y'')$
   ii. missing $x$: $0 = F(y, y', y'')$

   **This is a common source of mistakes.** In the first you make $y' = p(x)$ and in the second you make $y' = p(y)$. Find the necessary substitutions for $y''$ in both cases. **Hint: they are not the same**

(b) Solve $y'' = (y')^2$ using both methods. How do the methods differ?

(c) Solve the equations below:
   
   i. $xy'' + y' = 4x$
   ii. $yy'' = 3(y')^2$

5. Question 5 is on the next page. Go do it now and then come back.

6. Practice all the techniques and identifying when to use them. Solve the following equations. Ones with $(\ast)$ have challenging integrals in their solutions but you should be able to start them. Implicit solutions are ok on ones with $(\clubsuit)$

   (a) $xy' = y(\ln x - \ln y)$
   (b) $(\ast)(e^x + e^{-x})y' = y^2$
   (c) $(\ast\spadesuit)y' = \sqrt{x + 2y}$
   (d) $(\spadesuit)y^3y' = (y^4 + 1)\cos x$
   (e) $(\heartsuit)y' = \frac{y - x}{y + x}$
   (f) $(\ast\heartsuit)yy'' + (y')^2 = yy'$
   (g) $3y + x^4y' = 2xy$
   (h) $y' = x^2 - 2xy + y^2$

7. Bernoulli equations: $y' + P(x)y = Q(x)y^n$ for $n \neq 0, 1$

   (a) What happens when you make the substitution $v = y^{1-n}$? [Make the substitution]
   (b) Solve $xy' + 6y = 3xy^{4/3}$.

8. More of the same concepts

   (a) What happens with the substitution $v = \ln y$ in the differential equation $y' + P(x)y = Q(x)y\ln y$?
   (b) Solve $xy' - 4x^2y + 2y\ln y = 0$

**An Ending Thought:** *The difference between failure and success is doing a thing nearly right and doing a thing exactly right.*

– Edward Simmons
5. An alien has taken the table below and cleared all the entries. He was chased by the intergalactic police who at least got him to drop the table entries but they have no idea how to help put them back. They have hired you to restore the table to its original state.

<table>
<thead>
<tr>
<th>Name</th>
<th>General Form</th>
<th>How to Recognize</th>
<th>Substitution or Technique</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F(y/x) = 0$</td>
<td>Reducible</td>
<td>can factor</td>
<td>$y$ only appears as $y^1$</td>
</tr>
<tr>
<td>Homogeneous</td>
<td>Use $v = \frac{y}{x}$</td>
<td>Magic Substitution</td>
<td>Missing dependent variable $y$ in 2nd order equation</td>
</tr>
<tr>
<td>$y' + P(x)y = Q(x)$</td>
<td>Use $v = \alpha(x, y)$ and transform equation so you have $x, v, v'$</td>
<td>Every term has same degree</td>
<td></td>
</tr>
<tr>
<td>$F(x, y', y'') = 0$</td>
<td>Reducible</td>
<td>Separable</td>
<td>$F(y, y', y'') = 0$</td>
</tr>
<tr>
<td>Separable variables and integrate</td>
<td>Linear</td>
<td>$y$ appears inside of a function</td>
<td>Missing independent variable $x$ in 2nd order equation</td>
</tr>
<tr>
<td>$\frac{dy}{dx} = h(x)g(y)$</td>
<td>$F(\alpha(x, y)) = 0$</td>
<td>Integrating factors</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$p = y', y'' = pp'$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$p = y', p' = y''$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>