1. Solutions of Differential Equations

(a) Circle all those that could be the solution to a differential equation. Why or why not?

i. \( y = 4 \)  
ii. \( y^2 + x^2 = 1 \)  
iii. \( 4 \)  
iv. \( c_1 e^{3x} + c_2 e^{5x} = y \)  
v. \( \sin y + x + C = 0 \)  
vii. \( y = \frac{x}{2} \sin x \)  
viii. \( y = (2x^{4/3} + C)^{3/2} \)  
ix. \( \tan(xy) = C \)  

(b) Of the above, which are general solutions and which are particular solutions?

(c) In English, what is the difference and when do you need each?

(d) Which of the above are explicit solutions and which are implicit solutions?

(e) In English, what is the difference and when do you need each?

2. Separable Equations (Section 1.4 for more practice)

(a) Which of the following differential equations are separable?

i. \( y' = 6y^2x \)  
ii. \( x^3y' = x^2y - y^3 \)  
iii. \( \frac{dy}{dx} = 2x \sec y \)  
iv. \( 2\sqrt{xy}' = \cos^2 y \)  

(b) In English, what makes a differential equation separable?

(c) In English, how do you solve a separable equation?

(d) Solve the separable equations above. Give explicit solutions when possible.

3. Linear equations (Section 1.5 for more practice)

(a) Which of the following differential equations are linear?

i. \( x \frac{dy}{dx} - y = x \)  
ii. \( xy' + y = 3xy \)  
iii. \( 2xyy' = 4x^2 + 3y \)  
iv. \( (x^2 + 1) \frac{dy}{dx} = 6xe^{-\frac{3}{2}x^2} - 3x^3y \)  

(b) In English, what makes them linear?

4. Integrating Factors: solving linear equations relies on undoing the product rule

(a) Fill in the blanks that make the following statements true. Note that \( y \) is a function of \( x \).

i. \( \frac{d}{dx}(y \underline{\quad}) = 2xy + \underline{\quad}y' \)

ii. \( \frac{d}{dx}(y \underline{\quad}) = \underline{\quad}y + (\sin x)y' \)
iii. \( \frac{d}{dx}(y) = e^x y + xe^x \)

(b) i. What is the derivative of \( e^{\int Q(x) dx} \)?

ii. \( \frac{d}{dx}(y) = \ldots y + \ldots Q(x)y' \)

5. Put the steps to solving a linear first-order differential equation.

___ Collapse the LHS to the derivative of a product
___ Write the equation in the form \( P(x)y' + Q(x)y = R(x) \)
___ Use any initial condition
___ Add \( C \)
___ Recognize the equation as linear by the \( y^1 \)
___ Integrate both sides
___ Make the LHS look like the product rule in one of two ways:
   (a) Recognize an easy integrating factor to multiply by
   (b) Divide by \( P(x) \) [whatever is in front of \( y' \)], calculate \( \rho = e^{\int Q(x)/P(x) dx} \), multiply by \( \rho \).
___ Solve for \( y \)

6. Choosing an integrating factor

\[ y' - (\tan x)y = 1 \]

(a) Think about how to make the LHS look like a product rule. [Hint: what is \( \tan x \)?]
(b) Calculate \( \rho \) for the above differential equation.
(c) Which is easier?

7. Below are some differential equations. Some are separable. Some are linear. Some aren’t either. Solve those you can. Some are both.

(a) \( 3x^5y^2 + x^3y' = 2y^2 \)
(b) \( y' = 1 + x + y + xy \)
(c) \( xy' + 2y = 6x^2\sqrt{y} \)
(d) \( y^2y' + 4x^2 + y^2 = 0 \)
(e) \( xy' = x^2 - 2y - x + 1 \)
(f) \( (x^2 + 4)y' + 3xy = x \)

8. Solve linear equations from question 3.

An Ending Thought: If at first you do succeed try not to look too surprised.

– Unknown