MATH 115: Worksheet 5

September 23, 2014

Today we will be discussing **functions** and particularly **how to graph** functions

1. ALWAYS label your axes. Otherwise you will be eaten by a dinosaur. It might not happen immediately, but it will happen. You don’t want to risk it.

2. Determine which of the following are graphs of functions.

   (a) What test are you using?  
   (b) Why does it work?

3. Describe in words what each of the following shifts does. If you are unsure think about what happens with $f(x) = x^2$.

   (a) $f(x) - 4$  
   (b) $\frac{1}{2} f(x)$  
   (c) $f(x + 3)$  
   (d) $f(-x + 2)$  
   (e) $2f(x)$  
   (f) $6 - f(x)$

4. A graph of $f(x)$ and then four transformations follow. Match each with its function.

   (a) $f(x) + 2$  
   (b) $f(x) - 2$  
   (c) $f(x + 2)$  
   (d) $f(x - 2)$

5. A graph of $h(x)$ and then two transformations follow. Match each with its function.

   (a) $h(-x)$  
   (b) $-h(x)$
6. Graph each of the following functions using your knowledge of transformations. Describe the transformation at each step.

(a) \( y = x^2 \)  
(b) \( y = (x + 3)^2 \)  
(c) \( y = -(x + 3)^2 \)  
(d) \( y = 2 - (x + 3)^2 \)

7. Given the graph of function \( f(x) \) below, sketch the graph of \( y = -2f(-x) + 2 \).

HINT: break it into steps like the above problem.

8. If \( f(x) = a_nx^n + \ldots + a_0x^0 + c \), what are the conditions of \( a_i, b_i \), and \( c \) that ensure \( f(x) \) will be a polynomial function?

9. Which of the following functions are polynomials?

(a) \( f(x) = \frac{3x^2 + 25x + 2}{4} \)  
(b) \( g(x) = 3x^2 + 25x + 2 \)  
(c) \( h(x) = x^{-3} + 14x^{-2} - 6x + 101 \)  
(d) \( j(x) = \frac{x + 4}{x + 12x^2} \)  
(e) \( k(x) = (3 + \sqrt{x})(3 - \sqrt{x}) \)  
(f) \( l(x) = (x^{1/3} + 4)(x^{3/2} - 4) \)

10. Just like we took limits of sequences, we can also take limits of functions! One particular type of limit we are interested is as \( x \to \infty \) and \( x \to -\infty \) which describes the end behavior of the function.

(a) Start with \( f(x) = x^3 \). Use its graph to determine the following end behavior, ie calculate \( \lim_{x \to \infty} f(x) \) and \( \lim_{x \to -\infty} f(x) \).

(b) Now consider \( g(x) = 2x^3 - 7x^2 - 4x + 12 \). Without graphing, determine \( \lim_{x \to \infty} g(x) \) and \( \lim_{x \to -\infty} g(x) \). If you aren’t sure, test big positive and negative values for \( x \).

(c) Based on this example, what is the only thing you have to consider about \( g(x) \) to determine the long-term behavior?

(d) Practice with these. Find the end behavior of both ends:

i. \( p(x) = -4x^4 + 5x^3 - 356 \)  
ii. \( q(x) = x^2 - 10x^3 + 32x^3 \)  
iii. \( r(x) = (3x^2 - 2)(x^2 + 1)(\pi - x) \)

An Ending Thought: Don’t let the fear of striking out hold you back.

– Babe Ruth