MATH 220  
Mock Test 2  
Fall 2013

Name______________________________  NetID____________________

- Sit in your assigned seat (circled below).
- Circle your TA discussion section.
- Do not open this test booklet until I say START.
- Turn off all electronic devices and put away all items except a pen/pencil and an eraser.
- Remove hats and sunglasses.
- You must show sufficient work to justify each answer.
- While the test is in progress, we will not answer questions concerning the test material.
- Do not leave early unless you are at the end of a row.
- Quit working and close this test booklet when I say STOP.
- Quickly turn in your test to me or a TA and show your Student ID.

> AD3, TR 1:00-2:50, Cara Monical

FRONT OF ROOM – 314 Altgeld Hall
1. (5 points) Find $f'(x)$ given that $f(x) = 3x^3 + 2\tan x - 6\csc x + 3e^x + 2\ln x$

$$f'(x) = 90x^2 + 2\sec^2 x + 6\csc x \cot x + 3e^x + \frac{2}{x}$$

2. (5 points) Find $f'(x)$ given that $f(x) = \sin(x^2 + \ln(x) + e)$

$$f'(x) = \cos(x^2 + \ln(x) + e) \left( 2x + \frac{1}{x} \right)$$

3. (5 points) Find $f'(x)$ given that $f(x) = \frac{e^{4x} + 1}{x^3 + 2}$

$$f'(x) = \frac{(x^3 + 2) (4e^{4x}) - (e^{4x} + 1)(3x^2)}{(x^3 + 2)^2}$$

4. (5 points) Find $f'(x)$ given that $f(x) = x^3 \arcsin x$

$$f'(x) = x^3 \left( \frac{1}{\sqrt{1 - x^2}} \right) + 3x^2 \arcsin x$$
5. (5 points) Find $f'(x)$ given that $f(x) = \sec^3(\ln(\cos(3x)))$

$$f'(x) = 3\sec^2(\ln(\cos 3x)) \sec(\ln(\cos 3x)) \tan(\ln(\cos 3x)) \left(-\frac{\sin(3x) (3)}{\cos 3x}\right)$$

6. (5 points) Find $\frac{dy}{dx}$ given that $y = x^{\tan x}$

$$\ln y = (\tan x) (\ln x)$$

$$\frac{y'}{y^2} = \tan x \left(\frac{1}{x}\right) + \sec^2 x (\ln x)$$

$$y' = x^{\tan x} \left[\tan x \left(\frac{1}{x}\right) + \sec^2 x (\ln x)\right]$$
7. (10 points) Find the equation of the line tangent to the curve \( x \sin y + y^2 = x - 1 \) at the point \((1, 0)\). Write your simplified answer in the form \( y = mx + b \).

\[
x(\cos y \ y') + \sin y + 2y \ y' = 1
\]

\[
1 \ (\cos 0) \ y' + \sin (0) + 2(0) y' = 1
\]

\[
y' = 1
\]

\[
y - 0 = 1 \ (x - 1)
\]

\[
y = x - 1
\]

8. (5 points) The graph of \( y = f(x) \) has the property that the slope of the tangent line at the point \((x, y)\) is equal to \(-y\). If the graph goes through the point \((0, 2)\), then find a formula for \( f(x) \).

\[
\frac{dy}{dx} = -y
\]

\[
f(x) = Ce^{-x}
\]

\[
2 = Ce^0
\]

\[
C = 2
\]

\[
f(x) = 2e^{-x}
\]
9. (15 points) The left hand column contains the graphs of 5 functions. The right-hand column contains the graphs of their first derivatives. Match each function with its derivative.
10. (10 points) Evaluate the following limits.

(a) \( \lim_\limits{x \to 0} \frac{\cos(2x) - 1 + 2x^2}{x^4} \to 1 - 1 + 0 = 0 \)

\[ \lim_\limits{x \to 0} \frac{-2\sin(2x) + 4x}{x^3} \to 0 \]

\[ \lim_\limits{x \to 0} \frac{-4\cos(2x) + 4}{12x^2} \to 0 \]

\[ \lim_\limits{x \to 0} \frac{8\sin 2x}{24x} \to 0 \]

\[ \lim_\limits{x \to 0} \frac{16\cos(2x)}{24} = \frac{16}{24} = \frac{2}{3} \]

(b) \( \lim_\limits{x \to \infty} \left(1 - \frac{1}{x}\right)^x \)

\[ = \lim_\limits{x \to \infty} \ln \left(1 - \frac{1}{x}\right)^x \]

\[ = \lim_\limits{x \to \infty} x \ln \left(1 - \frac{1}{x}\right) \]

\[ = \lim_\limits{x \to \infty} \frac{\ln \left(1 - \frac{1}{x}\right)}{1/x} \to 0 \]

\[ = \lim_\limits{x \to \infty} \frac{1}{x^2} \to 0 \]

\[ = \frac{1}{2} \]

\[ e^{\frac{1}{2}} \]
11. (10 points) A 15 foot ladder is resting against a wall. The bottom is initially 10 feet away from the wall and is being pushed towards the wall at a rate of \( \frac{1}{4} \) ft/sec. How fast is the top of the ladder moving up the wall 12 seconds after we start pushing?

\[
\frac{dx}{dt} = -\frac{1}{4} \text{ ft/sec}
\]

After 12 seconds, have changed

\[
(-\frac{1}{4})(12) = -3
\]

started at 10 feet so now \( x = 7 \)

\[
Y = \sqrt{15^2 - x^2}
\]

\[
\frac{dy}{dt} = \frac{1}{2} \left( 15^2 - x^2 \right)^{-\frac{1}{2}} (-2x) \left( \frac{dx}{dt} \right)
\]

\[
= -\frac{x}{\sqrt{225 - x^2}} \left( \frac{dx}{dt} \right) = \frac{-x}{\sqrt{225 - x^2} \cdot 4}
\]

\[
\frac{dy}{dt} \bigg|_{x = 7} = \frac{-7}{\sqrt{1225 - 49}} = \frac{7}{4\sqrt{1176}} \text{ ft/sec}
\]
12. (10 points) A function \( f(x) \) has first derivative \( f'(x) = x^2 + 3x - 10 \).

(a) On which intervals is \( f(x) \) increasing?

\[
\begin{align*}
\text{\( f \) is inc when \( f'(x) > 0 \)} \\
x^2 + 3x - 10 = 0 \\
(x + 5)(x - 2) = 0 \\
x = -5, x = 2 \\
\hspace{1em} \begin{array}{ccc}
+ & - & + \\
-5 & 2 & \\
\end{array}
\end{align*}
\]

Interval: \((-\infty, -5) \) and \((2, \infty)\)

(b) On which intervals is \( f(x) \) concave down?

\[
\begin{align*}
\text{\( f \) is conc. down when \( f''(x) < 0 \)} \\
f''(x) = 2x + 3 = 0 \\
x = -\frac{3}{2} \\
\hspace{1em} \begin{array}{ccc}
- & + \\
-\frac{3}{2} & \\
\end{array}
\end{align*}
\]

Interval: \((-\infty, -\frac{3}{2})\)
13. (10 points) For $x > 0$, a line goes through $(0,0)$ and a point on the curve of $y = 3x^4e^{-4x}$. Which value of $x$ gives the line with the largest slope?

$$m = \frac{3x^4e^{-4x} - 0}{x - 0} = 3x^3e^{-4x}$$

$$\frac{dm}{dx} = 9x^2(e^{-4x}) - 12e^{-4x}(x^3)$$

$$= 3x^2e^{-4x}[3 - 4x] = 0$$

$x = 0$ \quad $3 - 4x = 0$

$x = \frac{3}{4}$

\[\begin{array}{ccc}
0 & + & \frac{3}{4} \\
\hline
 & - & \\
\end{array}\]

Maximum @ $x = \frac{3}{4}$
Students – do not write on this page!

1. (5 points) __________________________

2. (5 points) __________________________

3. (5 points) __________________________

4. (5 points) __________________________

5. (5 points) __________________________

6. (5 points) __________________________

7. (10 points) __________________________

8. (5 points) __________________________

9. (15 points) __________________________

10. (10 points) _________________________

11. (10 points) _________________________

12. (10 points) _________________________

13. (10 points) _________________________

**TOTAL (100 points) ________________**