1. When comparing two functions, it’s often useful to talk about which one “grows faster.” One way to define this is to say that \( f(x) \) grows faster than \( g(x) \) if eventually (i.e., as \( x \) gets really big), \( f(x) > g(x) \).

\( 2^x \) is always faster than \( x^a \)

(a) According to this definition, which grows faster, \( x \) or \( 2^x \)? (Sketch a graph if you’re not sure).

(b) How about \( x \) or \( 2^x \)? Use this to figure out which grows faster, \( x^2 \) or \( 2^x \).

(c) How about \( x^a \) and \( 2^x \)?

(d) Is there any positive number \( a \) for which \( x \) grows faster than \( 2^x \)? How about for which \( x^a \) grows faster than \( 2^x \)?

(e) People often talk about *polynomial growth* versus *exponential growth*. Which is faster? Does it make sense to group these two concepts?

2. A biologist is researching a newly-discovered species of bacteria. At time \( t = 0 \) hours, she puts one hundred bacteria into a favorable growth medium. Six hours later, she measures 450 bacteria. Find an exponential function to model this growth. How many bacteria will there be after six more hours? After nine hours from the second measurement?

\[ y = 100(\sqrt[6]{4.5})^t, \text{ six more hours: } 2025 \text{ bacteria} \]

3. The Richter magnitude \( M \) of an earthquake is defined in terms of the energy \( E \) in Joules released by the earthquake, with \( \log_{10} E = 4.4 + 1.5M \). Find the energy for earthquakes with magnitudes 4, 5, and 6. For each increase in \( M \) of 1, how does \( E \) change?

\( E = 10^10.4, 10^11.9, 10^13.4 \), grows by a factor of \( 10^{1.5} \) each time
4. Below are some “properties” of logarithms and exponentials. Some of these are true and some are false. Determine which properties of exponentials are true. Then from there, use the fact that logarithm is the inverse of exponential to determine which properties of logarithms are true.

• \((a^x)^y = a^{xy}\) true
• \(a^0 = 1\) true
• \(a^{x+y} = a^xa^y\) true
• \((a^x)^y = a^{xy}\) false
• \(a^x - a^y = a^{x/y}\) false
• \(a^1 = a\) true
• \(a^{x+y} = a^x + a^y\) false
• \(a^{x-y} = a^{x/y}\) true
• \((ab)^x = a^x b^x\) true

• \(\log_a (x + y) = (\log_a x)(\log_a y)\) false
• \(\log_a 0 = 1\) false
• \(\log_a (x - y) = \frac{\log_a x}{\log_a y}\) false
• \(\log_a (xy) = \log_a x + \log_a y\) true
• \(\log_a 1 = 0\) true
• \(\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y\) true
• \(\log_a (x^r) = r \log_a x\) true
• \(\log a = 1\) true
• \(\log_a (rx) = (\log_a x)^r\) false

5. Without your calculator, simplify \(\frac{7}{8\sqrt[5]{5}} = \frac{\sqrt[5]{7}}{40}\), \(\ln(x + 2)\)

6. Solve the inequalities \(1 < e^{3x-1} \leq 2\) and \(1 - 2 \ln x < 3\).

7. \(\frac{1}{3} < x \leq \frac{\ln 2 + 1}{3}, x \geq \frac{1}{e}\)

8. For each of the following functions, find a formula for the inverse function.

   (a) \(f(x) = \ln(4 + \sqrt[3]{x})\)
   (b) \(f(x) = \frac{3+2x}{3-4x}\)
   (c) \(f(x) = 3\ln(4 + \sqrt[2]{2x+1})\)
   (d) \(f(x) = \frac{1}{x}\)
   (e) \(f^{-1}(x) = (e^x - 4)^3\)
   (f) \(f^{-1}(x) = \frac{5x-3}{4x+2}\)
   (g) \(f^{-1}(x) = \frac{1}{2}[(e^{x/3} - 4)^5 - 1]\)
   (h) \(f^{-1}(x) = \frac{1}{x}\)

9. Are the trigonometric functions 1-1? Do they have inverses? What do we need to do to modify the trigonometric functions so they have inverses? Then, what are the domains and ranges of the following functions?

   Need to restrict the domain so they are 1-1 and therefore have inverses.

   (a) \(\arcsin(x), \text{ domain: } [-1,1], \text{ range } [-\frac{\pi}{2}, \frac{\pi}{2}]\)
(b) $\cos^{-1}(x)$, domain: [-1,1], range $[0,\pi]$
(c) $\arctan(x)$, domain: $\mathbb{R}$, range $(-\frac{\pi}{2}, \frac{\pi}{2})$
(d) $\csc^{-1}(x)$, domain: $|x| \geq 1$, range $(0, \frac{\pi}{2}] \cup [\pi, \frac{3\pi}{2})$
(e) $\sec^{-1}(x)$, domain: $|x| \geq 1$, range $[0, \frac{\pi}{2}) \cup (\pi, \frac{3\pi}{2}]$
(f) $\cot^{-1}(x)$, domain: $|x| \geq 1$, range $(0, \pi)$

10. Simplify the following. It may be helpful to draw a picture.

(a) $\sin(\arcsin(\frac{3}{5}))$, $\frac{3}{5}$
(b) $\cos(\arcsin(\frac{3}{5}))$, $\frac{4}{5}$
(c) $\tan(\sec^{-1}(\frac{7}{2}))$, $\frac{\sqrt{45}}{2}$
(d) $\sin(\arctan x)$, $\frac{x}{\sqrt{x^2+1}}$
(e) $\cos(\arcsin x)$, $\sqrt{1-x^2}$
(f) $\sec(\arccos x)$, $\frac{1}{x}$

11. Discuss with your group the most important concepts from Chapter 1. We are moving into Chapter 2 so make sure you understand everything in Chapter 1.

An Ending Thought: *Don’t let the fear of striking out hold you back.*

– Babe Ruth