1. Sketch the exponential function \( f(x) = b^x \).

2. Prove that any exponential function, \( f(x) = b^x \) where \( b > 0 \), is 1-1 on its entire domain.

3. Deduce that any exponential function, \( f(x) = b^x \), has an inverse. Then define \( g(x) = \log_b(x) \) as the inverse function to \( f(x) \). Suppose that \( y = \log_b(z) \), what can you say about \( b^y \)?

4. Solve each equation for \( x \):
   
   \[
   \begin{align*}
   (a) \quad & \ln(2x - 1) = 3 \\
   (b) \quad & e^x = 4 \\
   (c) \quad & \log_x 32 = 5 \\
   (d) \quad & e^{4x} = 3 \\
   (e) \quad & x^2 \ln x - 9 \ln x = 0 \\
   (f) \quad & x e^{-2x} + 2 e^{-2x} = 0 \\
   (g) \quad & 2 \ln(3x) = 1 \\
   (h) \quad & \ln(e^{2x}) = 6 \\
   (i) \quad & e^{4x} + 3 e^{2x} - 10 = 0
   \end{align*}
   \]

5. Use the definition of the logarithm and the properties of exponents to prove the following properties of the logarithm.
   
   \[
   \begin{align*}
   (a) \quad & \log_b 1 = 0 \\
   (b) \quad & \log_b b^c = c \\
   (c) \quad & b^{\log_b a} = a \\
   (d) \quad & \log_c(ab) = \log_c a + \log_c b \\
   (e) \quad & a \log_b c = \log_b (c^a)
   \end{align*}
   \]

6. Give three functions \( f(x), g(x), \) and \( h(x) \) such that the following functions can be written as \( f \circ g \circ h(x) \)
   
   \[
   \begin{align*}
   (a) \quad & \left[\tan^{-1}(4x + 5)\right]^3 \\
   (b) \quad & \sin^4(\ln x) \\
   (c) \quad & \frac{23}{\sqrt{e^x + 4}}
   \end{align*}
   \]

7. Given \( f(x) \) and \( g(x) \) below, find the domain of the composite function, \( f \circ g(x) \)
   
   \[
   \begin{align*}
   (a) \quad & f(x) = \ln(5 - x), g(x) = \sqrt{x - 30} \\
   (b) \quad & f(x) = \sqrt{x}, g(x) = 2 - \ln x \\
   (c) \quad & f(x) = \sqrt{x + 9}, g(x) = x^2
   \end{align*}
   \]

8. Find the domain of the following functions.
   
   \[
   \begin{align*}
   (a) \quad & \sqrt{x + 9} + \sqrt{x + 2} \\
   (b) \quad & \ln(5 - \sqrt{12 - 4x}) \\
   (c) \quad & \frac{8 - x}{\ln(x - 4)} \\
   (d) \quad & \frac{\sqrt{3x}}{\ln(6 - 0.5x)} \\
   (e) \quad & \frac{\sqrt{9 - x^2} + \ln(x + 1) + e^{x-2}}{\sqrt{x - 4}}
   \end{align*}
   \]

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9. A biologist is researching a newly-discovered species of bacteria. At time $t = 0$ hours, she puts one hundred bacteria into a favorable growth medium. Six hours later, she measures 450 bacteria. Find an exponential function to model this growth. How many bacteria will there be after six more hours? After nine hours from the second measurement?

10. Given an acute angle $\theta$ for which $\sin \theta = \frac{1}{4}$, evaluate the following quantity.

$$\sin(\pi - \theta) + \sin \theta + \cos(\pi - \theta) + \cos \theta$$

11. When comparing two functions, it’s often useful to talk about which one “grows faster.” One way to define this is to say that $f(x)$ grows faster than $g(x)$ if eventually (i.e., as $x$ gets really big), $f(x) > g(x)$.

(a) According to this definition, which grows faster, $x$ or $2^x$? (Sketch a graph if you’re not sure).
(b) How about $x$ or $2^{\frac{x}{2}}$? Use this to figure out which grows faster, $x^2$ or $2^x$.
(c) How about $x^3$ and $2^x$?
(d) Is there any positive number $a$ for which $x$ grows faster than $2^{\frac{x}{a}}$? How about for which $x^a$ grows faster than $2^x$?
(e) People often talk about polynomial growth versus exponential growth. Which is faster? Does it make sense to group these two concepts?

12. Discuss with your group what kinds of questions will be on the quiz next Tuesday and finish up anything remaining on the previous worksheets.

An Ending Thought: Enthusiasm moves the world.  

– Arthur Balfour