1. When we estimate distances from velocity data, it is sometimes necessary to use times $t_0, t_1, \ldots$ that are not equally spaced. We have still estimate distances using the time periods $\delta t_i = t_i - t_{i-1}$. For example, on May 7, 1992, the space shuttle Endeavour was launched on mission STS-49, the purpose of which was to install a new perigee kick motor in an Intelsat communications satellite. The table, provided by NASA, gives the velocity data for the shuttle between liftoff and the jettisoning of the solid rocket boosters. Use these data to estimate the height of the earth’s surface of the Endeavour, 62 seconds after liftoff.

<table>
<thead>
<tr>
<th>Event</th>
<th>Time (s)</th>
<th>Velocity (ft/ s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Launch</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Begin roll maneuver</td>
<td>10</td>
<td>185</td>
</tr>
<tr>
<td>End roll maneuver</td>
<td>15</td>
<td>319</td>
</tr>
<tr>
<td>Throttle to 89%</td>
<td>20</td>
<td>447</td>
</tr>
<tr>
<td>Throttle to 64%</td>
<td>32</td>
<td>742</td>
</tr>
<tr>
<td>Throttle to 104%</td>
<td>59</td>
<td>1325</td>
</tr>
<tr>
<td>Maximum dynamic pressure</td>
<td>62</td>
<td>1445</td>
</tr>
<tr>
<td>Solid rocket booster separation</td>
<td>125</td>
<td>4151</td>
</tr>
</tbody>
</table>

2. For the function $f$ whose graph is shown above, list the following quantities in increasing order, from smallest to largest, and explain your reasoning.

(a) $\int_0^8 f(x) \, dx$  
(b) $\int_0^3 f(x) \, dx$  
(c) $\int_3^8 f(x) \, dx$  
(d) $\int_4^8 f(x) \, dx$  
(e) $f'(1)$

3. Sketch the region whose area is given by the following integrals ($a$ and $r$ are positive constants). Then use geometry to evaluate the integral.

(a) $\int_{-a}^a 4 \, dx$  
(b) $\int_{-a}^a (a - |x|) \, dx$  
(c) $\int_0^2 (2x + 5) \, dx$  
(d) $\int_{-r}^r \sqrt{r^2 - x^2} \, dx$

4. Use the limit definition to evaluate the following integrals.

(a) $\int_4^{10} 6 \, dx$  
(b) $\int_1^3 3x^2 \, dx$  
(c) $\int_1^2 (x^2 + 1) \, dx$

5. Your grandmother asks you what you are learning in your favorite class (which is obviously this one). How would you explain the following to her?

(a) If $f(x) \geq 0$, then $\int_a^b f(x) \, dx \geq 0$
(b) If $f(x) \geq g(x)$, then $\int_a^b f(x) \, dx \geq \int_a^b g(x) \, dx$.
(c) If $m \leq f(x) \leq M$, then $m(b - a) \leq \int_a^b f(x) \, dx \leq M(b - a)$.

6. Use the above inequalities to verify:

$$\frac{\sqrt{2}\pi}{24} \leq \int_{\pi/6}^{\pi/4} \cos x \, dx \leq \frac{\sqrt{3}\pi}{24}$$
7. Find a (reasonable) upper and lower bound for the following integrals.

(a) \( \int_{-2}^{3} 3x^4 - 4x^3 - 12x^2 + 1 \, dx \)  
(b) \( \int_{-1}^{2} x\sqrt{4 - x^2} \, dx \)

8. A car braked with a constant deceleration of 16 ft/s², producing skid marks measuring 200 ft before coming to a stop. How fast was the car traveling when the brakes were first applied?

9. **Even and Odd Functions:** These questions will require you to think (crazy, I know). However, learning these quick checks can make your life a lot easier.

(a) Recall, what does it mean for a function to be even? What does it mean for a function to be odd? Think about the algebraic and geometric interpretations of this!

(b) Sketch an odd function like \( \sin x \). Use this to find the value of \( \int_{-\pi}^{\pi} \sin x \, dx \). How about \( \int_{-a}^{a} f(x) \, dx \) where \( f(x) \) is an even function?

(c) Sketch an even function like \( x^2 \). Use this to find the value of \( \int_{-2}^{2} x^2 \, dx \) in terms of \( \int_{0}^{2} x^2 \, dx \). How about \( \int_{-a}^{a} f(x) \, dx \) where \( f(x) \) is an odd function?

(d) Given that \( \int_{3}^{0} f(x) \, dx = 4, \int_{4}^{6} f(x) \, dx = 5, f(x) = 2 \) for \( 3 \leq x \leq 4 \), and that \( f(x) \) is an even function, what is \( \int_{-6}^{6} f(x) \, dx \)?

10. **Again with the thinking** Consider that \( f \) is continuous on the interval \([-5, 5]\) and \( \int_{0}^{5} f(x) \, dx = 4 \). Evaluate the following integrals.

(a) \( \int_{0}^{5} [f(x) + 2] \, dx \)  
(b) \( \int_{-2}^{3} f(x + 2) \, dx \)  
(c) \( \int_{-5}^{5} f(x) \, dx \) (\( f \) is even)  
(d) \( \int_{-5}^{5} f(x) \, dx \) (\( f \) is odd)

11. **You really should always think** Evaluate the following:

\[ \int_{\ln e^2}^{\sqrt{3}} \sin x^3 \cos \ln \frac{1}{x} \, dx \]

12. **There must be an easier way** There is— you are learning it (I think) tomorrow.

(a) Use the limit definition to evaluate \( \int_{a}^{b} x^2 \, dx \) (treat \( a \) and \( b \) as constants)

(b) Find an antiderivative of \( x^2 \). Call this \( F(x) \).

(c) Calculate \( F(b) - F(a) \). Does this look familiar?

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**An Ending Thought:** *A goal is a dream with a deadline.*

– Napoleon Hill