1. A function $f$ is defined on the entire real line. Suppose $n$ is the number of absolute extrema of a function $f$. What values could $n$ be? Draw a picture to represent each situation. 0, 1, 2

2. What two conditions are necessary to guarantee that a function has an absolute extrema on an interval? How do you find the absolute extrema on an interval? Practice with the functions and intervals below.

(a) $f(x) = x\sqrt{1-x}$, $[-1,1]$ max $2/3\sqrt{1/3}$, min $-\sqrt{2}$
(b) $f(x) = x^2e^{-x}$, $[-1,3]$, max $e$, min 0

3. This question discusses what a function and its derivatives can tell you about the shape of a graph. Fill out the following table:

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4. Use the above tests to sketch the curves of

(a) $y = \frac{x^2}{x^2+3}$
(b) $y = (4 - x^2)^5$
(c) $y = x + \cos x$

5. Suppose $g'(x) = \frac{8e^x(x^2-25)(x-4)^10}{\ln(x^2+9)}$. Find all critical numbers of $g(x)$. At each critical number, state whether $g(x)$ has a local maximum, local minimum, or neither at that point.

critical point -5 (max), 4 (neither), 5 (min)

6. Suppose $f'(x) = e^{2x}(x^2 + 25)(x - 3)^2(x^2 - 64)(2x - 1)$. Where $f$ is decreasing? (Give your answer in interval notation)

decreasing $(-\infty, -8), (1/2, 3), (3, 8)$

7. Let $f(x) = 8 + 5xe^{-3x}$

(a) The function $f$ is decreasing on the interval: $(1/3, \infty)$
(b) The function $f$ is increasing on the interval: $(-\infty, 1/3)$
(c) The function $f$ is concave up on the interval: $(2/3, \infty)$
(d) The function \( f \) is concave down on the interval: \((-\infty, 2/3)\)

8. Find two nonnegative numbers whose sum is 9 and so that the product of one number and square of the other number is a maximum.

3 and 6

9. We are going to build a rectangular pen with three parallel partitions using 500 feet of fencing. What dimensions will maximize the total area of the pen?

62.5 ft by 125 ft

10. An open (no lid) rectangular box with square base is to be made from 48 ft\(^2\) of material. What dimensions will result in a box with the largest possible volume?

4 ft x 4 ft x 2 ft

11. A container in the shape of a right circular cylinder with no top has surface area 3\(\pi\)ft\(^2\). What height \( h \) and base radius \( r \) will maximize the volume of the cylinder?

\( r = 1\text{ ft}, h = 1\text{ ft}\)

12. A sheet of cardboard 3ft by 4ft will be made into a box by cutting equal-sized squares from each corner and folding up the four edges. What will be the dimensions of the box with largest volume?

height is going to be \( \frac{7 - \sqrt{13}}{6} \), the other two sides are \( 4 - 2h \) and \( 3 - 2h \)

13. Suppose \( M \) is the largest slope on the graph of \( f(x) = \frac{3}{x^2 + 6} \). What is the value of \( M \)?

\( \frac{3}{16} \)

14. Consider all triangles formed by lines passed through the point \((8/9, 3)\) and both the \(x\) and \(y\) axes. Find the dimensions of the triangle with the shortest hypotenuse.

15. Find the point \((x, y)\) on the graph of \( y = \sqrt{x} \) nearest the point \((4, 0)\). \((7/2, \sqrt{7}/2)\)

16. A cylindrical can is to hold \( 20\pi \)m\(^3\). The material for the top and bottom costs \$10/m\(^2\) and material for the costs \$8/m\(^2\). Find the radius \( r \) and height \( h \) of the most economical can.

\( r = 2m, h = 5m \)

17. Harry Potter is standing on the east bank of a slow-moving river which is one mile wide and wishes to return to his campground on the west bank of the river. He can swim at 2 mph and walk at 3 mph. He must first swim across the river to any point on the opposite bank. From there he will walk to the campground, which is one mile north of the point directly across the river from where he currently is. What route will take the least amount of time?

swim to a point \( \frac{4}{\sqrt{5}} \) mi north of current point

18. We are constructing a window in the shape of a semi-circle over a rectangle. If the distance around the outside of the window is 12 feet, what dimensions will result in the rectangle having largest possible area?

19. Find the point where the curves \( y = x^3 - 3x + 4 \) and \( y = 3(x^2 - x) \) are tangent to each other, that is, have a common tangent line.

tangent line to both \( y - 6 = 9(x - 2) \), point is \((2, 6)\)