1. In the theory of electrical circuits, **Ohm’s law** describes the relationship between the voltage $V$ across a resistor, the electrical current $I$ passing through the resistor, and a quantity $R$ known as the resistance. The law can be written as follows:

$$V = IR$$

Usually voltage is measured in volts, current is measured in amperes (amps), and resistance is measured in ohms, where 1 ohm = 1 volt/amp. In a circuit with variable resistance, the quantities $V$, $I$, and $R$ might all depend on time.

(a) Take the derivative of Ohm’s law to find an equation relating $\frac{dV}{dt}$, $\frac{dI}{dt}$, and $\frac{dR}{dt}$.

(b) Suppose that the current is increasing at a rate of 0.3 amps/sec, while the resistance is holding steady at 4 ohms. How quickly is the voltage across the resistor increasing?

Now suppose that the voltage across the resistor is held constant at 20 volts, while the resistance is steadily increased at a rate of 0.4 ohms/sec.

(c) What is the current through the resistor when the resistance reaches 10 ohms?

(d) At what rate is the current changing at that time? Is it increasing or decreasing?

2. When expanding adiabatically (i.e. without gaining or losing heat), the pressure $P$ and volume $V$ of a sample of argon gas are related by the formula

$$PV^{5/3} = \text{constant.}$$

(a) Take the derivative of the above equation to find a formula relating $P$, $V$, $\frac{dP}{dt}$ and $\frac{dV}{dt}$.

(b) Suppose the volume of the sample is increasing at a rate of 0.4 L/s. How quickly is the pressure decreasing when the volume is 2.7 L and the pressure is 80 kPa?

3. A positively charged particle is flying in the vicinity of a charged conductor. The electric potential energy of the particle is given by the formula

$$E = k_e \frac{qQ}{r},$$

where $q$ is the charge of the particle, $Q$ is the charge on the conductor, $r$ is the distance between them, and $k_e = 0.90 \text{ cm} \cdot \text{J}/\mu\text{C}^2$.

(a) Assuming $q$ and $Q$ are constant, find a formula for $\frac{dE}{dt}$ in terms of $q$, $Q$, $k_e$, $r$ and $\frac{dr}{dt}$.

(b) At a certain instant, a particle with a charge of 1.5 $\mu\text{C}$ is 20 cm away from a conductor, and is flying directly towards the conductor at a rate of 2.0 cm/s. Given that the conductor has a charge of 4.0 $\mu\text{C}$, how quickly is the electrical potential energy of the particle increasing?

4. In physics, the energy stored in a stretched spring is determined by the equation

$$E = \frac{1}{2} k x^2,$$

where $E$ is the energy, $k$ is a constant (the **spring constant**), and $x$ represents the distance that the spring has been stretched.

(a) Find a formula for $\frac{dE}{dt}$ in terms of $k$, $x$, and $\frac{dx}{dt}$. 

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(b) A spring with spring constant \( k = 0.20 \text{ Joules/cm}^2 \) is being stretched at a rate of 1.5 cm/sec. How quickly is the energy stored in the spring increasing at the moment that \( x = 10 \text{ cm} \)?

5. In chemistry and physics, **Boyle’s Law** describes the relationship between the pressure and volume of a fixed quantity of gas maintained at a constant temperature. The law states that:

\[
PV = \text{a constant}
\]

where \( P \) is the pressure of the gas, and \( V \) is the volume.

(a) Take the derivative of Boyle’s law to find an equation relating \( \frac{dP}{dt} \), \( \frac{dV}{dt} \), \( P \), and \( V \).

(b) A sample of gas is placed in a cylindrical piston. At the beginning of the experiment, the gas occupies a volume of 250 cm\(^3\), and has a pressure of 100 kPa. The piston is slowly compressed, decreasing the volume of the gas at a rate of 50 cm\(^3\)/min. How quickly will the pressure of the gas initially increase?

6. In physics, the kinetic energy of a moving object is given by the formula

\[
K = \frac{1}{2}mv^2
\]

where \( m \) is the mass and \( v \) is the velocity. Suppose an object with a mass of 2.00 kg is accelerating at a rate of 6.00 m/s\(^2\). How quickly is the kinetic energy of the object increasing when the velocity is 23.0 m/s?

7. In the theory of electrical circuits, **Joule’s law** describes a relationship between the power \( P \) dissipated by a resistor, the electrical current \( I \) passing through the resistor, and the resistance \( R \). The law can be written as follows:

\[
P = I^2R.
\]

Usually power is measured in watts, current is measured in amps, and resistance is measured in ohms, where 1 ohm = 1 watt/amp\(^2\).

(a) Find an equation for \( \frac{dP}{dt} \) in terms of \( I \), \( R \), \( \frac{dI}{dt} \), and \( \frac{dR}{dt} \).

(b) Suppose that the current through a resistor is increasing at a rate of 0.04 amps/sec, while the resistance is increasing at a rate of 5 ohms/sec. How quickly is the power dissipated by the resistor increasing when the current is 0.3 amps and the resistance is 60 ohms?

8. The **ideal gas law** relates the temperature, pressure, and volume of an ideal gas. Given one mole of gas, the pressure \( P \), volume \( V \), and temperature \( T \) are related by the equation

\[
PV = RT
\]

where \( R \) is the **molar gas constant** \((R \approx 8.314 \text{ kPa} \cdot \text{liters/kelvin})\).

(a) Suppose that one mole of ideal gas is held in a closed container with a volume of 25 liters. If the temperature of the gas is increased at a rate of 3.5 kelvin/min, how quickly will the pressure increase?

(b) Suppose instead that the temperature of the gas is held fixed at 300 kelvin, while the volume decreases at a rate of 2.0 liters/min. How quickly is the pressure of the gas increasing at the instant that the volume is 20 liters?

**An Ending Thought:** You can’t fall if you don’t climb. But there’s no joy in living your whole life on the ground.

– Unknown