1 Chain Rule Practice

1. Differentiate the following functions

(a) \( y = (3x + 1)^2 \)

(b) \( y = \sqrt{13x^2 - 5x + 8} \)

(c) \( y = (1 - 4x + 7x^5)^{30} \)

(d) \( y = (4x + x^{-5})^{\frac{1}{3}} \)

(e) \( y = \left(\frac{8x - x^6}{x^3}\right)^{-\frac{3}{5}} \)

(f) \( y = \sin(5x) \)

(g) \( y = e^{5x^2+7x-13} \)

(h) \( y = e^{\cot x} \)

(i) \( y = 3 \tan \sqrt{x} \)

(j) \( y = \cos^2(x^3) \)

(k) \( y = \frac{1}{5} \sec^4(4 + x^3) \)

(l) \( y = e^{\cos^5(3x^4)} \)

(m) \( y = \tan^3 \sqrt{\cot(7x)} \)

2. Assume that \( h(x) = f(g(x)) \), where both \( f \) and \( g \) are differentiable functions. If \( g(-1) = 2 \), \( g'(-1) = 3 \), and \( f'(2) = -4 \), what is the value of \( h'(-1) \)?

3. Assume that \( h(x) = f(x)^3 \), where \( f \) is a differentiable function. If \( f(0) = \frac{1}{2} \) and \( f'(0) = \frac{8}{3} \), determine an equation of the line tangent to the graph of \( h \) at \( x = 0 \).

4. Explain the relationship between \( f' \) and \( g' \) given that \( g(x) = f(3x) \).

5. Explain the relationship between \( f' \) and \( g' \) given that \( g(x) = f(x^2) \).

6. The table below shows some values of the derivative unknown function \( f \). Complete the table by finding (if possible) the derivative of each transformation of \( f \). If it isn’t possible to determine a value, what value do you need?

(a) \( g(x) = f(x) - 2 \)

(b) \( h(x) = 2f(x) \)

(c) \( r(x) = f(-3x) \)

(d) \( s(x) = f(x + 2) \)

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2 Implicit Differentiation

1. Let us find the derivative of \( y = \tan^{-1}(x) \).

(a) Rewrite the equation so there are no inverse trig functions.
(b) Take the derivative of both sides with respect to \( x \). Since \( y \) is a function of \( x \), you will need to use the chain rule.
(c) Solve for \( y' \)
(d) In this equation, replace \( y \) with what it equals
(e) Simplify! (Hint: you may need to draw a triangle)
(f) Repeat the process to find the derivative of \( y = \arcsin(x) \) and \( y = \arccos(x) \).

2. Now let us try to find some tangent lines to the curve \( y^2 = x^3 - x \)

(a) Differentiate both sides of the equation with respect to \( x \), thinking of \( y \) as a function of \( x \) (you will need the chain rule again!)
(b) Solve for \( y' \) (in terms of both \( x \) and \( y \))
(c) Find an equation of the tangent line to this curve at \( (-\frac{1}{2}, \sqrt{\frac{3}{8}}) \).
(d) Find an equation of the tangent line to this curve at \( (-\frac{1}{2}, -\sqrt{\frac{3}{8}}) \).
(e) Find an equation of the tangent line to this curve at \( (0,0) \). What went wrong? (Hint: Check the graph)

3. Use the process above to find the derivative of \( y = \ln(x) \) given that you know the derivative of \( e^x \). This process is called implicit differentiation.

4. Find \( \frac{dy}{dx} \) by implicit differentiation.

(a) \( x^2 + y^2 = 36 \)  \hspace{1cm} (e) \( x^3 - 3x^2y + 2xy^2 = 12 \)
(b) \( \sqrt{x} + \sqrt{y} = 9 \)  \hspace{1cm} (f) \( \sin x + 2 \cos 2y = 1 \)
(c) \( x^3 - xy + y^2 = 4 \)  \hspace{1cm} (g) \( \sin x = x(1 + \tan y) \)
(d) \( x^3y^3 - y = x \)  \hspace{1cm} (h) \( y = \sin(xy) \)

5. Based on the examples above, when do you need to use implicit differentiation?

**An Ending Thought:** *Go to work. Do your best. Don’t outsmart your common sense.*

– “Love Like Crazy” by Lee Brice