Series worksheet.

Decide convergence/divergence. If convergent, is it absolute or conditional? Show your work and state any tests you use.

1. $\sum_{k=1}^{\infty} \frac{1}{k2^k}$ converges by comparison test w/ geometric series $\sum \frac{1}{2^k}$, since $\frac{1}{k2^k} < \frac{1}{2^k}$. $\left|\frac{1}{k2^k}\right| = \frac{1}{k2^k}$, so absolute

2. $\sum_{n=1}^{\infty} \frac{(-1)^n}{1+\sqrt{n}}$ converges by alt. series test: $\frac{1}{1+\sqrt{n}} > 0$, $\lim_{n\to\infty} \frac{1}{1+\sqrt{n}} = \lim_{n\to\infty} \frac{1/\sqrt{n}}{1/\sqrt{n}+1} = 0$
$\frac{1}{1+\sqrt{n+1}} < \frac{1}{1+\sqrt{n}}$.
conditional by limit comparison test w/ $\sum \frac{1}{n^{1/2}}$, since $\lim_{n\to\infty} \frac{n^{1/2}}{1+\sqrt{n}} = 1$.

3. $\sum_{n=2}^{\infty} \frac{\ln(n)}{n^2+2n+1}$ converges by limit comparison test w/ $\sum 1/n^{3/2}$ since $\lim_{n\to\infty} \frac{\ln(n)/n^2+2n+1}{1/n^{3/2}} = \lim_{n\to\infty} \frac{n^{3/2}\ln(n)}{n^2+2n+1} = 0$
absolute since $\left|\frac{\ln(n)}{n^2+2n+1}\right| = \frac{\ln(n)}{n^2+2n+1}$.

4. $\sum_{j=2}^{\infty} \left(\frac{(-1)^j}{j}\left(\frac{1}{\ln(j)}\right)^{\frac{3}{2}}\right)$ absolute convergence by integral test: $\left|\frac{(-1)^j}{j}\left(\frac{1}{\ln(j)^{3/2}}\right)\right| = \frac{1}{j\ln(j)^{3/2}} > 0$
and $\int_2^{\infty} \frac{dx}{x(\ln(x))^{3/2}} = \int_{\ln 2}^{\infty} \frac{du}{u^{3/2}}$ converges.

5. $\sum_{k=1}^{\infty} \frac{(-1)^k}{1+100^{-k}}$ diverges by n^{th} term test $\lim_{k\to\infty} \frac{(-1)^k}{1+100^{-k}}$ does not exist.

6. $\sum_{n=4}^{\infty} \underbrace{\frac{10n^4+13n^3-1490n^2-n-1}{231n^6-n^4-32n-21}}_{a_n}$ converges absolutely by limit comparison test: for n very large, terms are positive and $\lim_{n\to\infty} a_n/\frac{1}{n^2} = \frac{10}{231}$.

7. $\sum_{k=3}^{\infty} \frac{k!(2k)!}{(3k)!}$ ratio test: $\frac{(k+1)!(2(k+1))!}{(3(k+1))!} \cdot \frac{(3k)!}{k!(2k)!} = \frac{(k+1)(2k+2)(2k+1)}{(3k+3)(3k+2)(3k+1)} \xrightarrow[k\to\infty]{} \frac{4}{27} < 1$
so converges absolutely

8. $\sum_{j=1}^{\infty} \frac{1\cdot 3\cdot 5\cdot \ldots \cdot (2j-1)}{j!}$
ratio test: $\frac{1\cdot 3\cdot 5\cdots(2j-1)(2j+1)}{(j+1)!} \cdot \frac{j!}{1\cdot 3\cdot 5\cdots(2j-1)} = \frac{2j+1}{j+1} \to 2 > 1$
diverges

9. $\sum_{k=1}^{\infty} \frac{k^k}{3^{(k^2)}}$
root test: $\sqrt[k]{\left(\frac{k^k}{3^{(k^2)}}\right)} = \sqrt[k]{\left(\frac{k}{3^k}\right)^k} = \frac{k}{3^k} \to 0 < 1$ so converges

10. $\sum_{n=1}^{\infty} (\sqrt{n}-\sqrt{n-1})^n$
root test: $\sqrt[n]{(\sqrt{n}-\sqrt{n-1})^n} = \sqrt{n}-\sqrt{n-1} = \frac{(\sqrt{n}-\sqrt{n-1})(\sqrt{n}+\sqrt{n-1})}{\sqrt{n}+\sqrt{n-1}}$
$= \frac{n-(n-1)}{\sqrt{n}+\sqrt{n-1}} = \frac{1}{\sqrt{n}+\sqrt{n-1}} \to 0 < 1$

converges