

Techniques of Integration:

• substitution: $\int F(u(x)) u'(x) dx = \int F(u) du$
 [chain rule]

recognize this \uparrow
 $u = u(x)$
 $du = u'(x) dx$

• Int. by parts: $\int u dv = uv - \int v du$
 [product rule]

Geometry of \mathbb{C}

$e^{ix} = \cos x + i \sin x$
 $e^{ix} e^{iy} = e^{i(x+y)}$ gives most trig identities.

• trig: (*) $\int \sin^n x \cos^m x dx$ } $\sin^2 x + \cos^2 x = 1$, $\sin^2 x = \frac{1}{2}(1 - \cos(2x))$
 $\cos^2 x = \frac{1}{2}(1 + \cos(2x))$

$\int \tan^n x \sec^m x dx$ } $\frac{m \text{ even}}{n \text{ odd}}$ } $1 + \tan^2 x = \sec^2 x$

$\frac{d}{dx} \tan x = \sec^2 x$, $\frac{d}{dx} \sec x = \sec x \tan x$

(*) substitution:

$\sqrt{a^2 - x^2}$ $\Rightarrow x = a \sin \theta$
 $\sqrt{a^2 + x^2}$ $\Rightarrow x = a \tan \theta$
 $\sqrt{x^2 - a^2}$ $\Rightarrow x = a \sec \theta$

• rational functions: (1) $\frac{P(x)}{Q(x)} = S(x) + \frac{R(x)}{Q(x)}$ $\deg(R) < \deg(Q)$ (long d. div.)

(2) Factor $Q(x)$: (luck, quad. formula)

\hookrightarrow linear & irreducible quadratic, $x^2 + bx + c$

(3) partial fraction: $(x-a)$

linear: $(x-a)^n$ factor in $Q(x) \Rightarrow \frac{A_1}{x-a} + \frac{A_2}{(x-a)^2} + \dots + \frac{A_n}{(x-a)^n}$

quadratic: $(x^2+bx+c)^n \dots \Rightarrow \frac{B_1x+C_1}{x^2+bx+c} + \dots + \frac{B_nx+C_n}{(x^2+bx+c)^n}$

(4) compute — linear clear
 — quad \Rightarrow complete the square & substitute

Approximate integration.

Midpoint, Trapezoidal, Simpson's rule.

$\Delta x(1, \dots, 1)$

$\frac{\Delta x}{2}(1, 2, 2, \dots, 2, 1)$

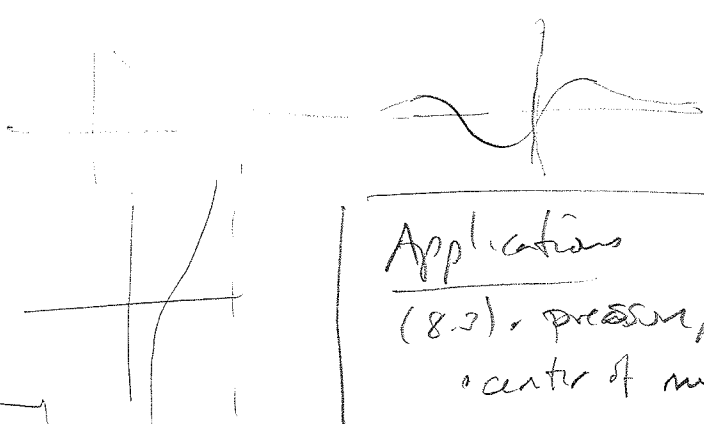
$\frac{\Delta x}{3}(1, 4, 2, 4, \dots, 2, 4, 1)$ and

mid

- don't need to know formulas for error bounds, but know how to use them

Improper Integrals

$$\int_a^b f(x) dx = \lim_{t \rightarrow \infty} \int_a^t f(x) dx$$



Comparison theorem

Applications

(8.3), pressure/force
center of mass

Curves

parametric, polar
(8.1, 2, 10.1-10.4)

- tangent lines
- area
- arc length
- surface area (revolution)

$$\int 2\pi(\text{distance to line}) \sqrt{x'^2 + y'^2} dt$$

Sequences & series :

- def'n, convergence, abs/conditional (series)
- tests: integral, comparison, alt series test, ratio & root test

- Power series: interval/radius of convergence, functions, derivatives/integrals, Taylor series, remainder, estimating.