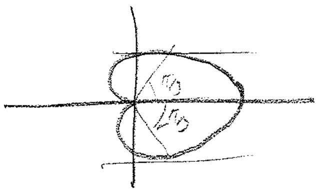
Math 271 4/22/11

tangents to polar curves:

EX When is the tangent line to cardiod $r = 1+\cos\theta$ horizontal?



$$f(\theta) = 1+\cos\theta$$

$$m(\theta) = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}$$

$$= \frac{-\sin^2\theta + (1+\cos\theta)\cos\theta}{\sin\theta\cos\theta - (1+\cos\theta)\sin\theta}$$

$$= \frac{\cos^2\theta - \sin^2\theta + \cos\theta}{-1}$$

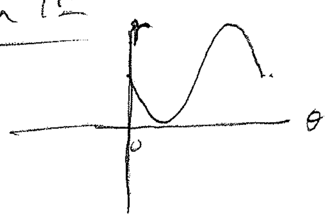
$$= \frac{\cos^2\theta - (1-\cos^2\theta) + \cos\theta}{-1} = \frac{2\cos^2\theta + \cos\theta - 1}{-1} = 0?$$

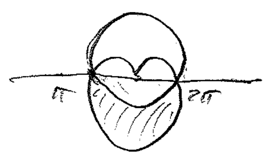
$$\cos\theta = \frac{-1 \pm \sqrt{1+8}}{4} = \frac{-1\pm 3}{4} = \frac{1}{2}, -1 \implies \theta = \pm\pi/3, \pi$$

pts: $(r,\theta) = (3/2, \pm\pi/3)$ & $(0,\pi)$

Area EX Find area inside $r = 1-\sin\theta$, outside $r=1$

Graph 1st





$$\int_{\pi}^{2\pi} \frac{1}{2}(1-\sin\theta)^2 d\theta -$$

Power series

- Defn's: $\sum_{n=0}^{\infty} c_n(x-a)^n$, $\{c_n\}_{n=0}^{\infty}$ sequence of real #'s, $a \in \mathbb{R}$ center.
- radius / interval of convergence

  $0 \leq R \leq \infty$

  - compute using series convergence tests – specifically ratio/root tests.
  - for interval, find R then test endpts (there, ratio & root will fail in general)

- for $x \in (a-R, a+R)$

  $f(x) = \sum_{n=0}^{\infty} c_n(x-a)^n$ sum of series

  defines a function f on $(a-R, a+R)$ "like a polynomial":

  - $f'(x) = \sum_{n=1}^{\infty} c_n n(x-a)^{n-1}$
  - $\int f(x)\,dx = \sum_{n=0}^{\infty} \frac{c_n}{n+1}(x-a)^{n+1} + C.$

  $\Big\}$ same radius of convergence.

  use this to find power series from series you know

  Eg: $\ln(1-x) = f(x) \Rightarrow f'(x) = \frac{-1}{1-x} = \sum_{n=0}^{\infty} -1\, x^n$

  $\Rightarrow f(x) = \sum_{n=0}^{\infty} \frac{-1}{n+1} x^{n+1} + C \qquad C = f(0) - \sum_{n=0}^{\infty} \frac{-1}{n+1}(0)^{n+1} = \ln(1) - 0 = 0$

  $\Rightarrow f(x) = \sum_{n=0}^{\infty} \frac{-1}{n+1} x^{n+1} = \sum_{n=1}^{\infty} \frac{-1}{n} x^n$

- Taylor & Maclaurin series:

  $f(x), a \rightsquigarrow \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$ ($a=0$ Maclaurin series)

  - radius of convergence? – see above
  - $f(x) \stackrel{?}{=} \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$? – look at remainder. $R_n(x) = f(x) - T_n(x)$

    $T_n(x) = n^{\text{th}}$ Taylor polynomial at $a = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$

    Taylor's Thm: $|f^{(n+1)}(x)| \leq M$ for $x \in (a-d, a+d)$

    then $R_n(x) \leq \frac{M}{(n+1)!}|x-a|^{n+1}$ for $x \in (a-d, a+d)$

  - Evaluate integrals w/ Taylor series

ex's e^x, $\sin x$, $\cos x$, $\frac{1}{1-x}$, $\tan^{-1}(x)$, $(1+x)^k$

Approximating function by taylor polynomials.

- error estimates via Taylor's Theorem or alt. series estimate.

Curves

- parametric curves. $x=f(t)$, $y=g(t)$
  - sketching
  - finding parametrization
  - eliminating parameter.
  - calculus
    - tangents
    - arc length
    - surface area
    - area
- polar coords.
  - polar vs. cartesian
  - curves
    - sketching
    - tangents
    - area
    - arc length.