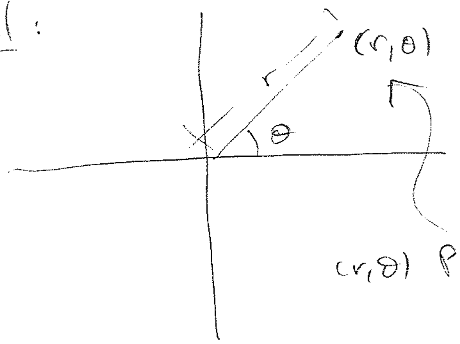
Polar coords & curves

Recall:



$x = r\cos\theta$ $\quad r^2 = x^2 + y^2$

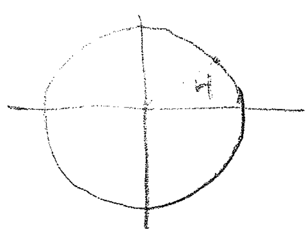
$y = r\sin\theta$ $\quad \tan\theta = \frac{y}{x}$

(r,θ) polar coords of point $\qquad$ cartesian coords (x,y)

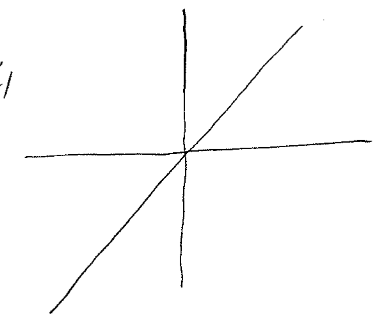
Not UNIQUE: [origin = $(0,\theta)$ for any θ.
$(r,\theta) = (-r, \theta+\pi) = (r, \theta+2\pi) \ldots$

Given an equation in r & θ, the set of points in the plane with at least one polar coord satisfying the equation is called the graph of the equation.

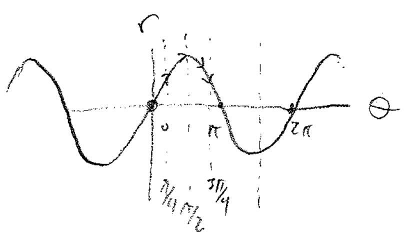
Ex $r = 4$

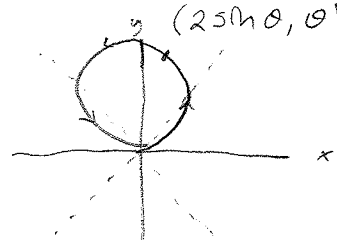


$\theta = \pi/4$



$r = 2\sin\theta$?





(r,θ)-polar plane

repeats after π since $2\sin(\theta+\pi) = -2\sin(\theta)$

look in (θ,r)-cartesian plane; just graph of the function

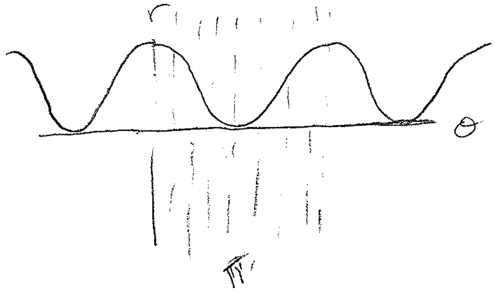
Here we can change to cartesian coords:

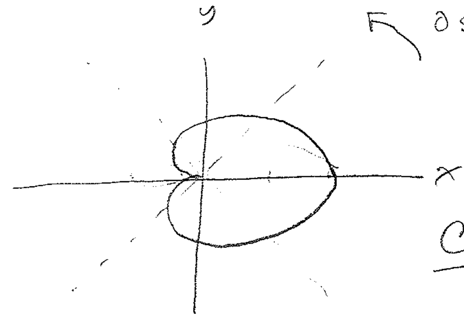
$$r = 2\sin\theta \Rightarrow r^2 = 2r\sin\theta \Rightarrow x^2+y^2 = 2y \Rightarrow x^2+y^2-2y+1 = 1$$
$$\Rightarrow x^2 + (y-1)^2 = 1$$

[not always easy, or useful]

Ex $r = 1 + \cos\theta$

use same idea





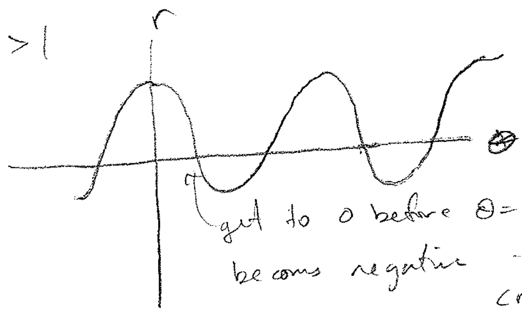
$0 \le \theta \le \pi$ distance to origin ≥ 0

Cardioid

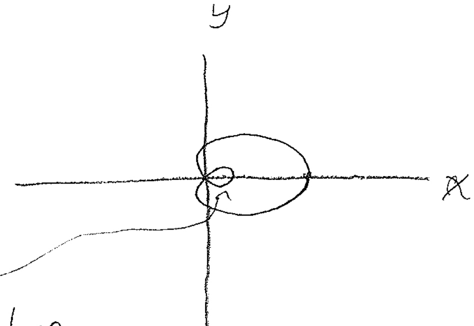
More generally

$r = 1 + d\cos\theta$

$d > 1$



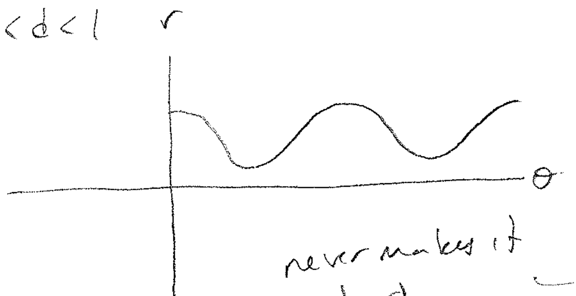
get to 0 before $\theta = \pi$. becomes negative



creates a loop.

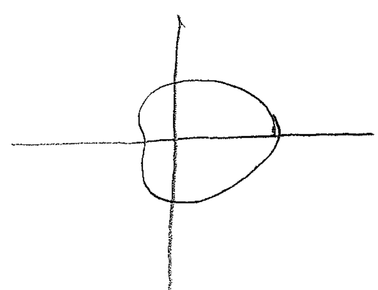
limaçons

$0 < d < 1$

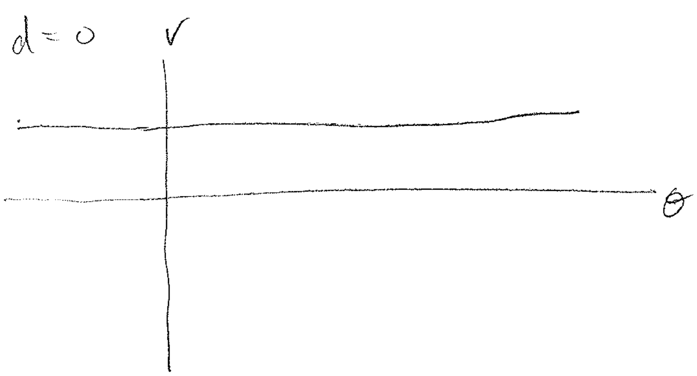


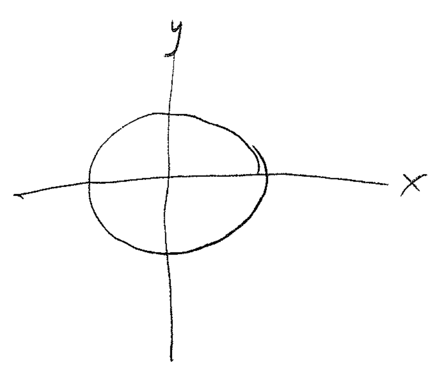
never makes it to 0

never gets to 0

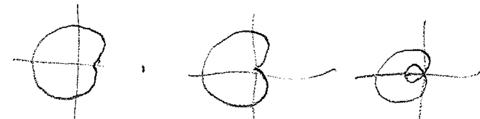


$d = 0$

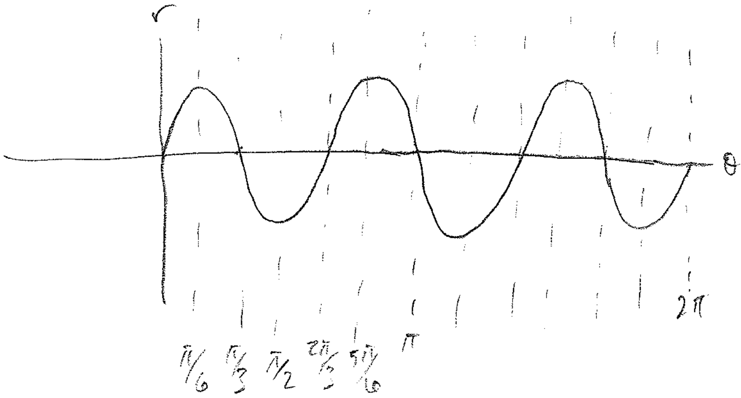




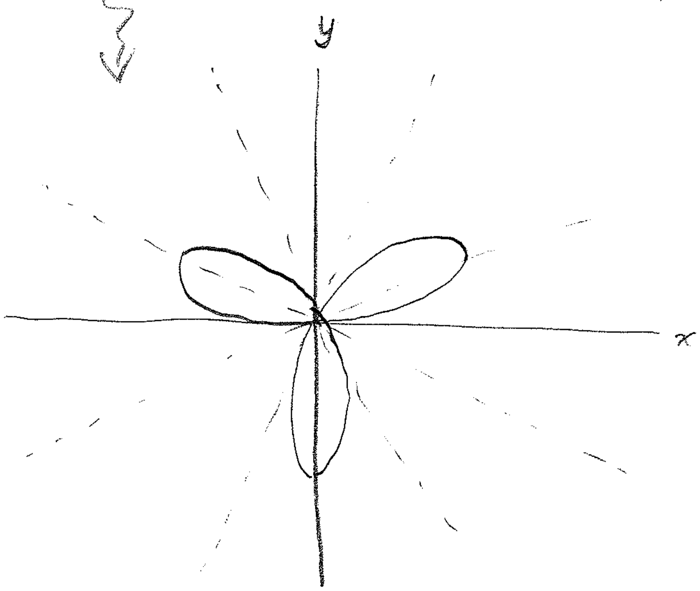
$d < 0$? — symmetric pictures

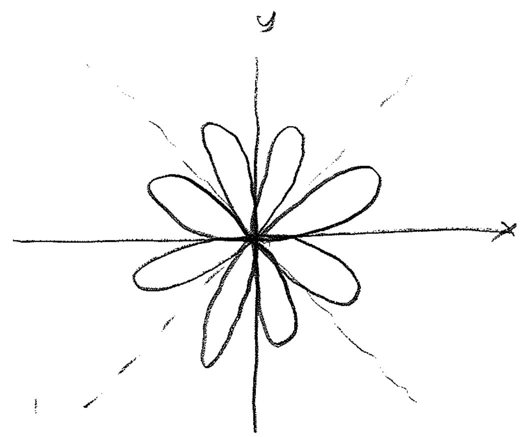


roses $\quad r = \sin(3\theta)$

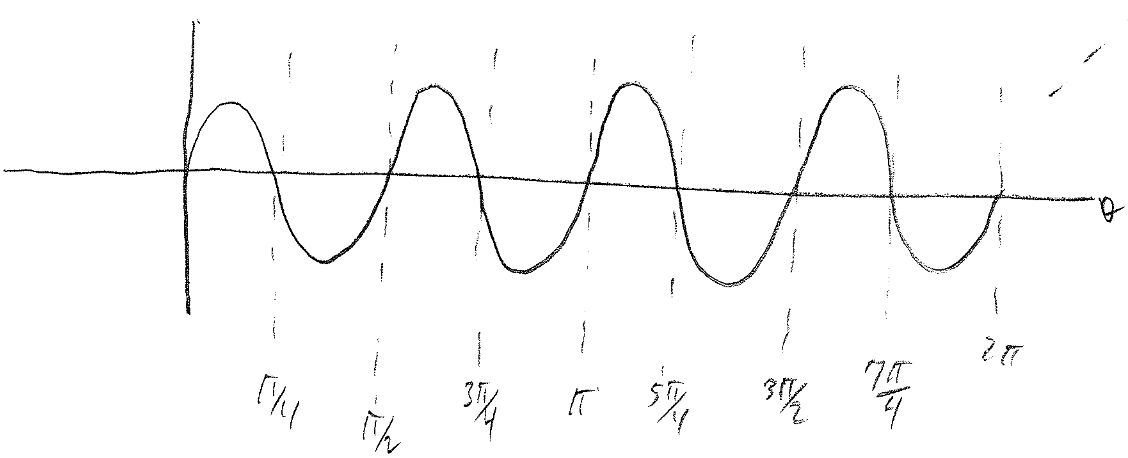


$\sin(3(\theta+\pi)) = \sin(3\theta+\pi) = -\sin(3\theta)$





$r = \sin(4\theta)$



Calculus:

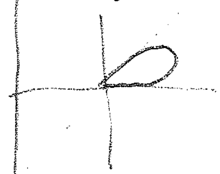
- Slopes of tangent lines to $r = f(\theta)$?

  treat it as a parametric curve:

  $$x = r\cos\theta = f(\theta)\cos\theta$$
  $$y = r\sin\theta = f(\theta)\sin\theta$$

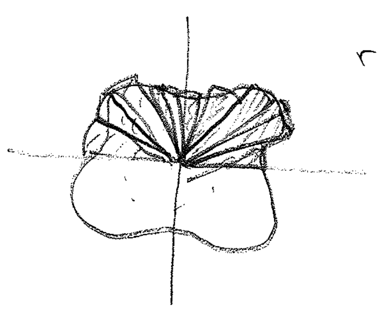
  $$\frac{y'}{x'} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}$$

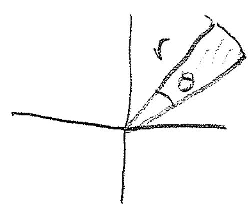
EX Length of one petal of rose $r = \sin 4\theta$.



$$\int_0^{\pi/4} \sqrt{16\cos^2 4\theta + \sin^2 4\theta}\, d\theta$$

- Areas bdd by polar curve $r = f(\theta)$

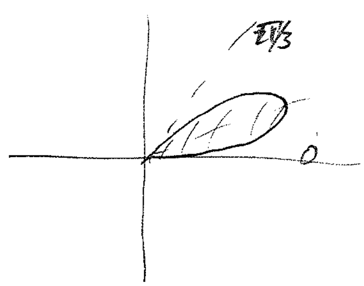




$$\text{Area} = \frac{\theta}{2\pi}(\pi r^2) = \frac{\theta}{2} r^2$$

$$\text{Area} \approx \sum_{i=1}^{n} \tfrac{1}{2}(f(\theta_i^*))^2 \Delta\theta \xrightarrow[n\to\infty]{} \int_{\theta_0}^{\theta_1} \tfrac{1}{2}(f(\theta))^2 d\theta$$

EX Area inside one petal of rose $r = \sin 3\theta$.



$$\text{Area} = \int_0^{\pi/3} \tfrac{1}{2}(\sin 3\theta)^2 d\theta$$

$$= \tfrac{1}{2}\int_0^{\pi/3} \sin^2 3\theta\, d\theta = \tfrac{1}{4}\int_0^{\pi/3} 1 - \cos(6\theta)\, d\theta$$

$$= \tfrac{1}{4}(\theta)\Big|_0^{\pi/3} = \pi/12.$$

- Arc length? $\displaystyle\int_{\theta_0}^{\theta_1} \sqrt{(x')^2 + (y')^2}\, d\theta = \int_{\theta_0}^{\theta_1} \sqrt{(f'(\theta))^2 + (f(\theta))^2}\, d\theta$

$$(f'(\theta)\cos\theta - f(\theta)\sin\theta)^2 + (f'(\theta)\sin\theta + f(\theta)\cos\theta)^2 = (f'(\theta))^2 + (f(\theta))^2$$