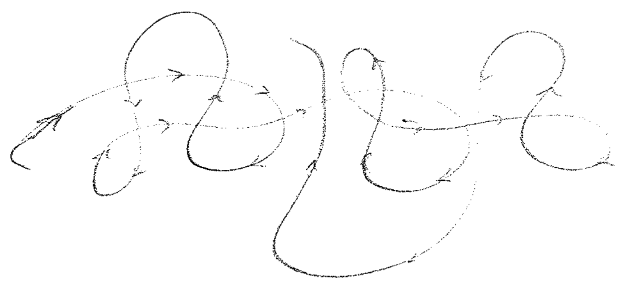
Math 231 4/11/11

Parametric curves

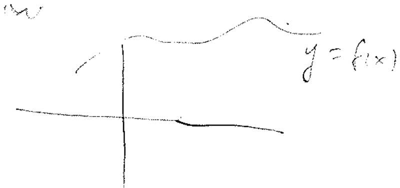
A particle moving in the plane traces out a curve



How can we describe this mathematically?

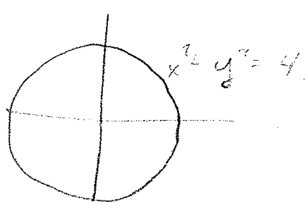
If the curve is "simple" we may be able to describe it by the graph of a function

ex

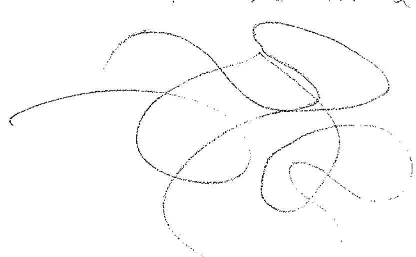


or the zero set of an equation

ex



May not be possible in a reasonable way



Moreover, this doesn't capture all the information

If a particle is traveling around a circle from 9:00AM to 9:02AM, we may want to know where it is at 9:01:42.536, and

$$(x-a)^2+(y-b)^2=r^2$$

doesn't tell us that..

We can introduce another variable, call it t, and then express the x & y coordinates of the particle at time t as functions of t:

$$x=f(t) \quad y=g(t)$$

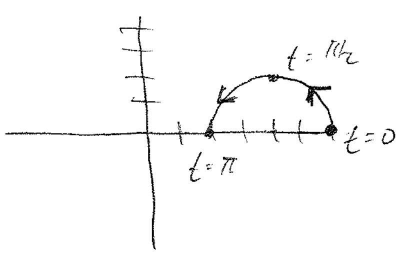
these are called <u>parametric equations</u> for the <u>parametric curve</u>. (or <u>parametrized curve</u>)

t is called the <u>parameter</u>.

[A parametrized curve need not describe the position of a particle, but this provides a nice geometric interpretation.]

<u>Ex</u> sketch & describe the parametric curve.

$$x=2\cos t+4, \quad y=2\sin t, \quad t\in[0,\pi]$$



observe that for every t

$$(x-4)^2+y^2=(2\cos t+4-4)^2+(2\sin t)^2$$
$$=4\cos^2 t+4\sin^2 t=4$$

so parametric curve is contained in circle centered at $(4,0)$ of radius 2.

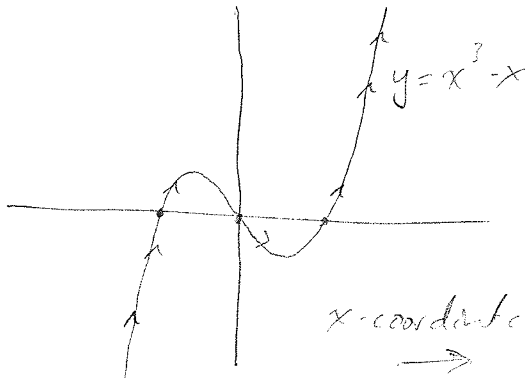
Its the upper semi circle traversed counterclockwise.

Ex [student] $x = t+1$, $y = t^3 + 3t^2 + 3t - t$. $t \in \mathbb{R}$.

$t = x - 1$ $y = (x-1)^3 + 3(x-1)^2 + 3(x-1) - (x-1)$

$= x^3 - 3x^2 + 3x - 1 + 3x^2 - 6x + 3 + 3x - 3 - x + 1$

$= x^3 - x$



x-coordinate increases as t does $\longrightarrow$

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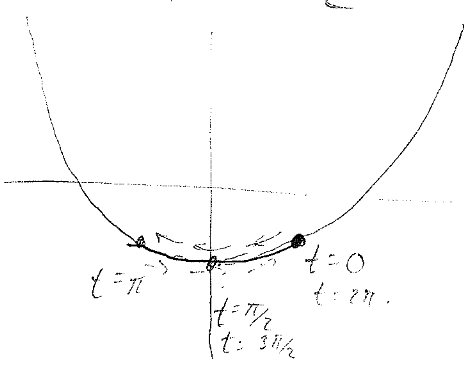
move to before 1st example

When the parameter is constrained to an interval, $a \le t \le b$, for the parametric curve defined by $x = f(t)$, $y = g(t)$ we call $(f(a), g(a))$ the <u>initial point</u> and $(f(b), g(b))$ the terminal point — in first example, $(6,0)$ = init point, $(2,0)$ = terminal point.

Ex: $x = \cos t$, $y = \cos^2 t - 2$ $t \in [0, 2\pi]$.

$x^2 - y = \cos^2 t - \cos^2 t + 2 = 2$

$y = x^2 - 2$



observe $-1 \le x \le 1$

$-2 \le y \le -1$

init $= (1,-1)$

term $= (1,-1)$