Another test for convergence/divergence similar to the ratio test is

Root Test Suppose $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L$. Then

1. If $L < 1$, then $\sum a_n$ converges absolutely
2. If $L > 1$ (or $\lim_{n\to\infty} \sqrt[n]{|a_n|} = \infty$), then $\sum a_n$ diverges
3. If $L = 1$, get no information.

Similar idea: eg. ① $\sqrt[n]{|a_n|} < r < 1$ for n sufficiently large, so $|a_n| < r^n$, compare w/ $\sum r^n$.

Useful when terms are n^{th} powers:

Ex $\displaystyle\sum_{n=1}^{\infty} \left(\frac{4n^2+2n-1}{5n^2+4}\right)^n$

Root test: $\displaystyle\lim_{n\to\infty} \sqrt[n]{\left|\left(\frac{4n^2+2n-1}{5n^2+4}\right)^n\right|} = \lim_{n\to\infty} \frac{4n^2+2n-1}{5n^2+4}$

$\displaystyle= \lim_{n\to\infty} \frac{4+\frac{2}{n}-\frac{1}{n^2}}{5+\frac{4}{n^2}} = \frac{4}{5} < 1$, so series converges

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Lots of tests / techniques for deciding convergence/divergence.

- n^{th} term test for divergence
- integral test
- comparison / limit comparison.
- alt series.
- ratio/root test.
- p-series.
- geometric series.