Math 231
1-31-11

More integration :-

last time, saw how to compute integrals of rational functions.

Some times integrals can be converted to that of rational function by subst.

Ex $\int \csc x\,dx = \int \frac{dx}{\sin x} = \int \frac{\sin x}{\sin^2 x}\,dx = \int \frac{\sin x}{1-\cos^2 x}\,dx \underset{u=\cos x}{=} \int \frac{-du}{1-u^2} = -\frac{1}{2}\left(\int \frac{du}{1-u} + \int \frac{du}{1+u}\right)$

(or $\int \sec x\,dx$)

$du = -\sin x\,dx$

$= -\frac{1}{2}\left(-\ln|1-u| + \ln|1+u|\right) + C$

$= \frac{1}{2}\left(\ln\left|\frac{1-u}{1+u}\right|\right) + C = \ln\sqrt{\frac{1-\cos x}{1+\cos x}} + C$

$$\boxed{\frac{1}{1-u^2} = \frac{A}{1-u} + \frac{B}{1+u} = \frac{(A+B) + (A-B)u}{1-u^2} \qquad A = B = \frac{1}{2}}$$

$$= \ln\sqrt{\frac{(1-\cos x)^2}{1-\cos^2 x}} + C = \ln\left|\frac{1}{\sin x} - \frac{\cos x}{\sin x}\right| + C = \ln|\csc x - \cot x| + C.$$

Ex $\int \frac{\sqrt[3]{x-2}}{x}\,dx = \int \frac{u\,3u^2\,du}{u^3+2} = \int \frac{3u^3}{u^3+2}\,du$
rational.

$u = \sqrt[3]{x-2}$
$u^3 = x-2$
$3u^2\,du = dx$

— Read §7.5 —

Worksheet

For time, just compute 6 as indefinite integral

Integration worksheet.

**Compute**

1. $\int \frac{e^{2t}}{1+e^{4t}}dt = \frac{1}{2}\int\frac{du}{1+u^2} = \frac{1}{2}\tan^{-1}(u)+C = \frac{1}{2}\tan^{-1}(e^{2t})+C$

$u = e^{2t}$

$du = 2e^{2t}dt$

2. $\int \frac{\ln(x)}{x\sqrt{1+(\ln(x))^2}}dx = \frac{1}{2}\int\frac{du}{\sqrt{u}} = \sqrt{u}+C = \sqrt{1+(\ln(x))^2}+C$

$u = 1+(\ln(x))^2$

$du = \frac{2\ln(x)}{x}dx$

3. $\int x\csc(x)\cot(x)dx = -x\csc x + \int \csc x\,dx = -x\csc x + \ln|\csc x - \cot x| + C$

$dv = \csc x\cot x\,dx \qquad v = -\csc x$

$u = x \qquad du = dx$

4. $\int \sin(\sqrt{t})dt = \int 2x\sin x\,dx = -2x\cos x + \int 2\cos x\,dx = -2x\cos x + 2\sin x + C$

$x = \sqrt{t}$

$x^2 = t \qquad 2x\,dx = dt \qquad u = 2x \qquad 2dx = du$

$dv = \sin x\,dx \qquad v = -\cos x$

$= -2\sqrt{t}\cos\sqrt{t} + 2\sin\sqrt{t} + C$

5. $\int \frac{\sqrt{2x-1}}{2x+3}dx = \int\frac{u^2}{u^2+4}du = \int\frac{u^2+4-4}{u^2+4}du = \int 1-\frac{4}{u^2+4}du = u - \frac{4}{2}\tan^{-1}\left(\frac{u}{2}\right)+C$

$u = \sqrt{2x-1}$

$u^2 = 2x-1 \qquad u^2+4 = 2x+3$

$2u\,du = 2dx$

$= \sqrt{2x-1} - 2\tan^{-1}\left(\frac{\sqrt{2x-1}}{2}\right)+C$

6. $\int_0^{\pi/4}\tan^5(\theta)\sec^3(\theta)d\theta = \int_0^{\pi/4}\tan^4\theta\sec^2\theta\tan\theta\sec\theta\,d\theta = \int_1^{\sqrt{2}}(u^2-1)^2u^2du = \int_1^{\sqrt{2}}u^6-2u^4+u^2du$

$u = \sec\theta$

$du = \sec\theta\tan\theta$

$= \frac{u^7}{7} - \frac{2u^5}{5} + \frac{u^3}{3}\Big|_1^{\sqrt{2}} = \sqrt{2}\left(\frac{8}{7}-\frac{8}{5}+\frac{2}{3}\right) - \left(\frac{1}{7}-\frac{2}{5}+\frac{1}{3}\right)$

7. $\int \frac{\tan^{-1}(x)}{x^2}dx = -\frac{\tan^{-1}x}{x} + \int\frac{dx}{x(1+x^2)} = -\frac{\tan^{-1}x}{x} + \int\frac{-x}{1+x^2}dx + \int\frac{dx}{x} = -\frac{\tan^{-1}x}{x} - \frac{1}{2}\ln(1+x^2) + \ln|x| + C$

$u = \tan^{-1}x \qquad du = \frac{1}{1+x^2}dx$

$dv = \frac{1}{x^2}dx \qquad v = -\frac{1}{x}$

$\frac{1}{x(1+x^2)} = \frac{Ax+B}{1+x^2} + \frac{C}{x} = \frac{Ax^2+Bx+C+Cx^2}{x(1+x^2)}$

$B = 0$

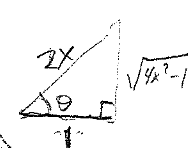
$C = 1$

$A+C = 0$

8. $\int \frac{dx}{x^2\sqrt{4x^2-1}} = \int\frac{\frac{1}{2}\sec\theta\tan\theta\,d\theta}{\frac{1}{4}\sec^2\theta\tan\theta} = 2\int\frac{d\theta}{\sec\theta} = 2\int\cos\theta\,d\theta = 2\sin\theta + C = 2\frac{\sqrt{4x^2-1}}{2x}+C = \frac{\sqrt{4x^2-1}}{x}+C$

$2x = \sec\theta$

$2dx = \sec\theta\tan\theta\,d\theta$




9. $\int \sqrt{1+e^x}dx = \int\frac{2u^2du}{u^2-1} = 2\int\frac{u^2-1+1}{u^2-1}du = 2\int 1+\frac{1}{u^2-1}du$

$= 2u + \int\frac{du}{u-1} - \int\frac{du}{u+1} = 2u + \ln|u-1| - \ln|u+1| + C$

$= 2u + \ln\left|\frac{u-1}{u+1}\right| + C$

$= 2\sqrt{1+e^x} + \ln\left(\frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}\right) + C$

$u = \sqrt{1+e^x}$

$u^2 = 1+e^x$

$2u\,du = e^x dx$

$\frac{2u}{u^2-1}du = dx$

$\frac{1}{u^2-1} = \frac{A}{u-1} + \frac{B}{u+1} = \frac{(A+B)u+(A-B)}{u^2-1}$

$A+B = 0 \qquad A-B = 1$