Math 231, Spring 2011
1/19/11

[Introductions / Course info] – review substitution §5.5.

$\mathbb{R}$ = real numbers – one drawback: polynomials, e.g. x^2+1, do not necessarily have roots.

Complex numbers: create a new number called i w/ $i^2=-1$ (so $i=\sqrt{-1}$)

$\mathbb{C} = \{a+bi \mid a,b \in \mathbb{R}\}$

addition and multiplication are defined by

$$(a+bi)+(c+di) = a+c+(b+d)i$$

$$(a+bi)(c+di) = ac-bd+(ad+bc)i$$

(for mult: treat i like a variable, then replace i^2 w/ -1)

Ex $z = 1+2i$, $w = \sqrt{3}+\pi i$

add & mult: $z+w$, zw $\Longrightarrow$ $z+w = (1+\sqrt{3})+(2+\pi)i$

$zw = \sqrt{3}-2\pi+(2\sqrt{3}+\pi)i$.

Honors problem explores $\mathbb{C}$ more — eg. every $z \in \mathbb{C}$, $z \neq 0$ has an inverse $\frac{1}{z}$.

of course $\mathbb{R} \subset \mathbb{C}$, as $a+0i$, for $a \in \mathbb{R}$

the imaginary numbers are numbers of the form $0+bi$, for $b \in \mathbb{R}$.

Q sums and products of real #s are real #s, what about imaginary #'s?

A No: $i \cdot i = -1$

Fundamental Theorem of Algebra: Any polynomial $p(x)$ of degree n w/ coeffs in $\mathbb{C}$ (or $\mathbb{R}$) has n roots (not necessarily distinct)

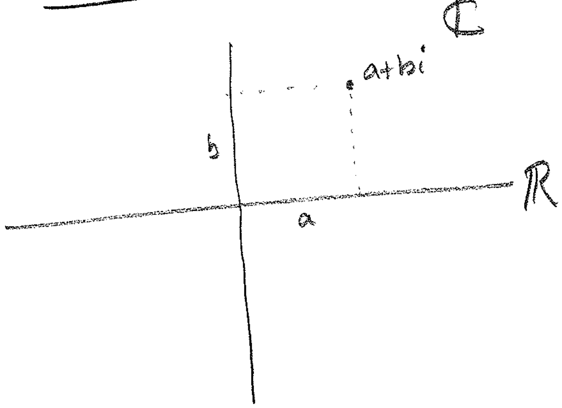
that is $p(x)$ can be factored into a product of linear factors.

Geometry of $\mathbb{C}$:

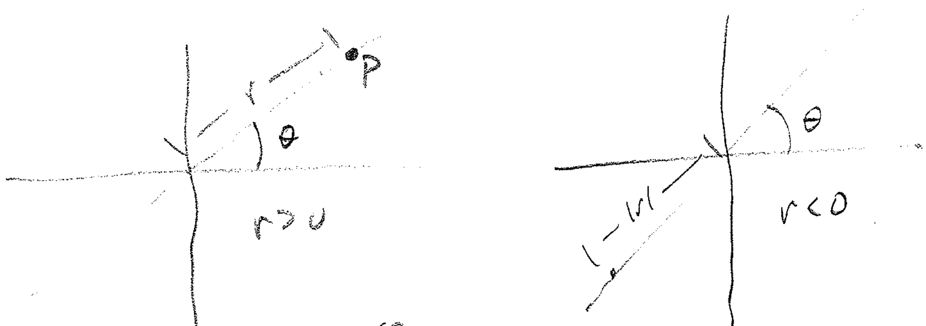
Recall that $\mathbb{R}$ is visualized as a line — the real line. $\mathbb{C}$ is pairs of real #'s, so can be visualized as a plane, — the complex plane

$a+bi \longleftrightarrow (a,b)$

$\mathbb{C} \longleftrightarrow \mathbb{R}^2$

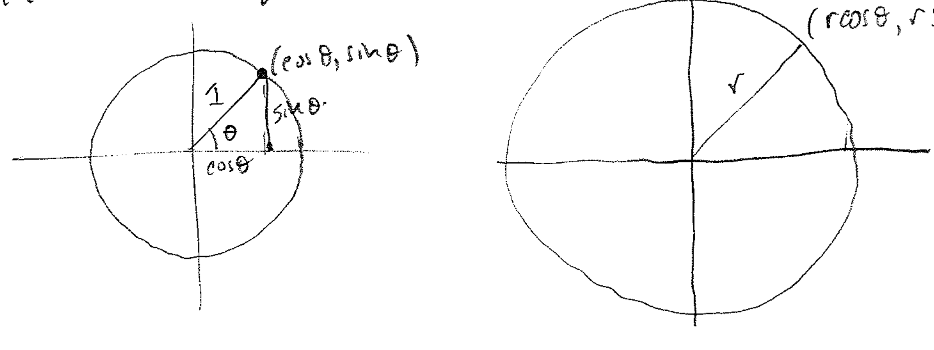


Given $P \in \mathbb{R}^2$, polar coordinates for the point are a pair of #s (r,θ) so that P lies on a line through the origin O w/ angle θ from +'ve x-axis (in radians) and signed distance r



(r,θ)

rel'n to rectangular (or Cartesian) coordinates



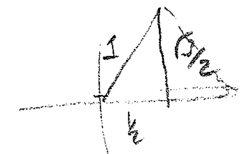
valid for any r, θ:

$x = r\cos\theta$

$y = r\sin\theta$

Ex Find rect. coords of point P w/ polar coords:

polar	rect
$(1, \pi/4)$	$(\cos \pi/4, \sin \pi/4) = (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$
$(-2, \pi/3)$	$(-2\cos \pi/3, -2\sin \pi/3) = (-1, -\sqrt{3})$



Find another polar coords!

Ex $(1, \pi/4) \rightsquigarrow (-1, 5\pi/4)$

$(-2, \pi/3) \rightsquigarrow (2, 4\pi/3) \rightsquigarrow (2, 10\pi/3) \rightsquigarrow (2, -2\pi/3)$

observe that polar (r, θ) is also related to (x,y) by:

$r^2 = x^2+y^2$, $\tan\theta = \frac{y}{x}$ (not a functional relationship of r, θ in terms of x, y)

Back to $\mathbb{C}$:

$z \in \mathbb{C}$ can be written as $z = r\cos\theta + i r\sin\theta = r(\cos\theta + i\sin\theta)$

We declare $e^{i\theta} := \cos\theta + i\sin\theta$. — satisfying justification later...

for now observe that any function $f: \mathbb{R} \to \mathbb{C}$ can be written as $f(x) = u(x) + iv(x)$, w/ $u, v: \mathbb{R} \to \mathbb{R}$. we can define f' and $\int f\,dx$ by $f'(x) = u'(x) + iv'(x)$ & $\int f(x)\,dx = \int u(x)\,dx + i\int v(x)\,dx$. — F.T.C. holds

Also, observe that for $f(x) = \cos x + i\sin x$ we have

$f'(x) = -\sin x + i\cos x = i(i\sin x + \cos x) = i f(x)$

hence $\frac{d}{dx}(e^{ix}) = ie^{ix}$ if w

We also write $e^{x+iy} = e^x(\cos y + i\sin y) = e^x e^{iy}$, then check $e^{z+w} = e^z e^w$