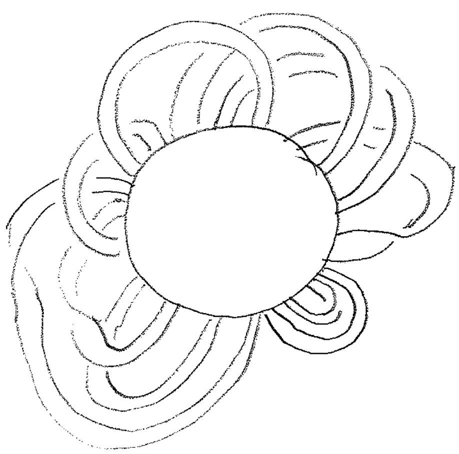
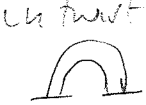
Sketch of proof: Given a surface S, consider a handle decomposition of S. - look 1st only at initial disk D & 1-handles.

Step 1 Apply handle slides so all 1-handles are attached to initial disk D - induct on the # of handles.



2 types of 1-handles

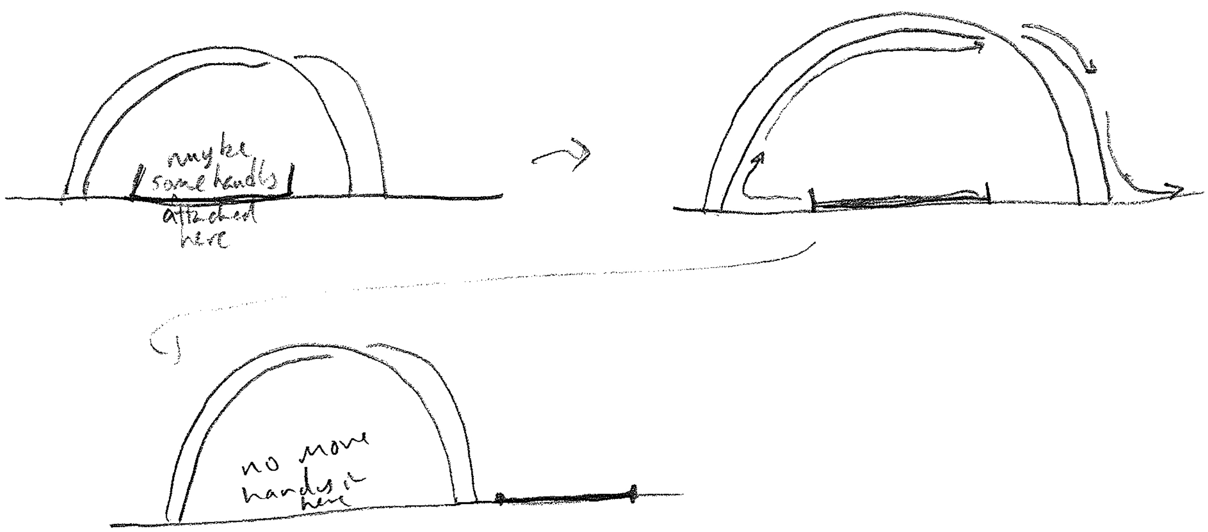
- untwisted



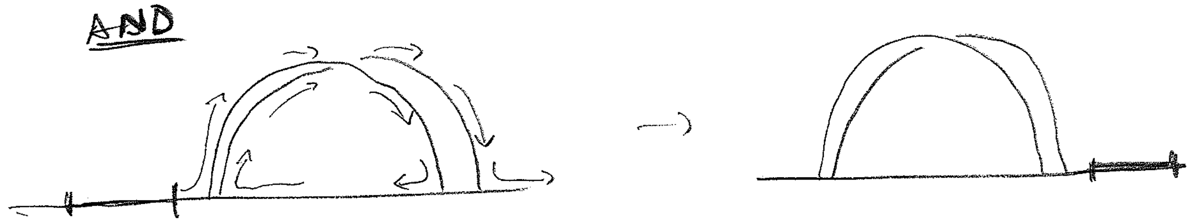
- twisted (once twisted)

give the circle & $[0,1]\times\{0\}$ and $[0,1]\times\{1\}$ directions - can be described in these terms

Step 2 apply handle slide to isolate all twisted 1-handles:

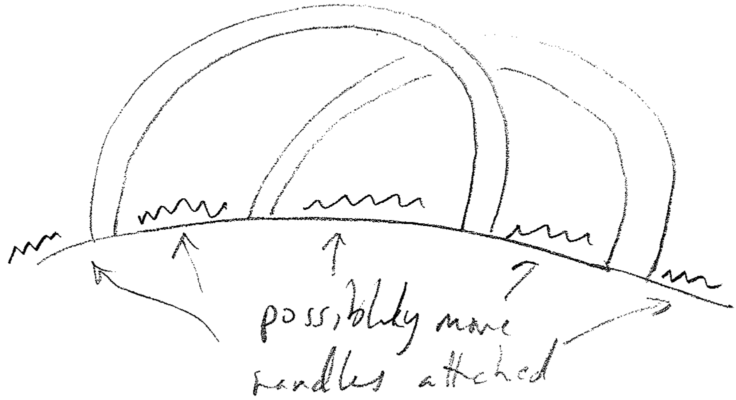


AND

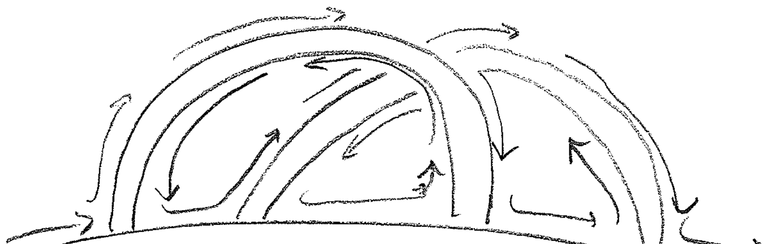


End up with

Step 3 A linked pair of 1-handles has form

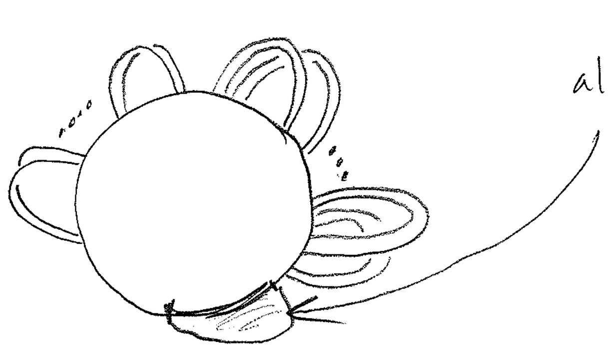


apply handle slides to isolate all linked pairs.

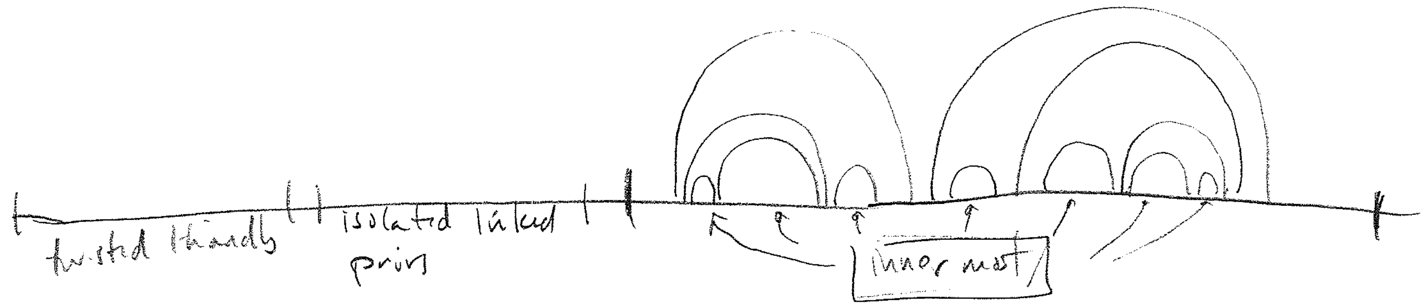


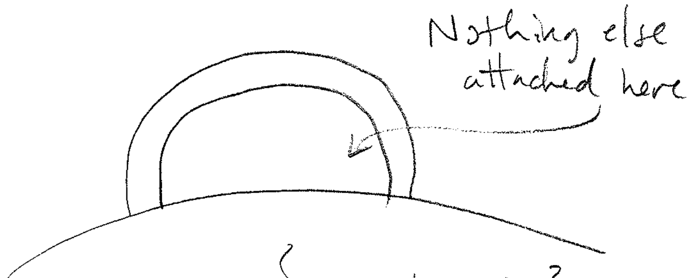
slide all other handles out

End up with



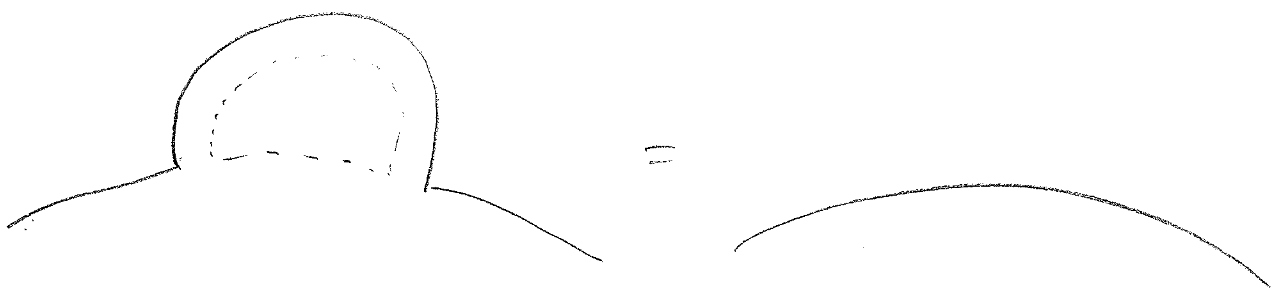
<u>Step 4</u> attach disks to un twisted unlinked pairs, one at a time along inner most 1-handles $\Longrightarrow$ "cancels" these 1-handles:
(draw with arcs w.l.o.g. — no twisting to worry about).





— glue in a disk

glue D^2



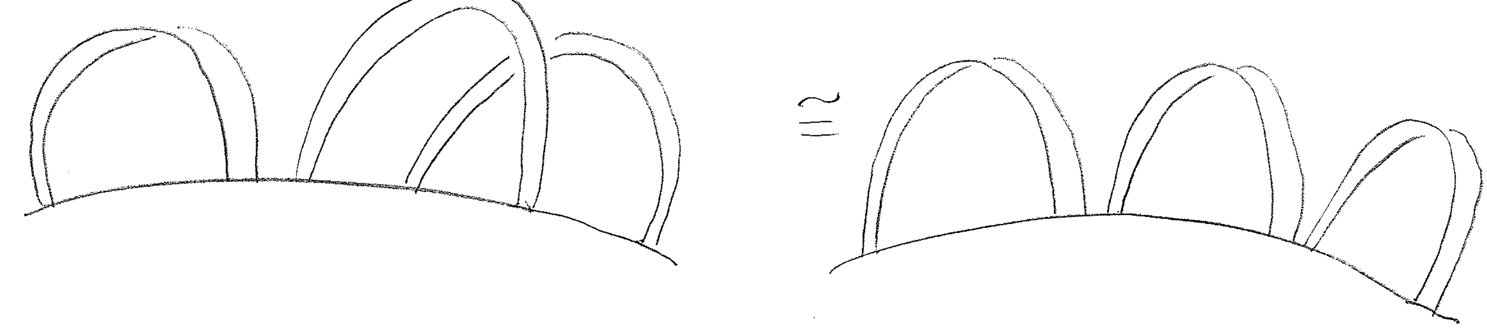
Inductively cancel all these 1-handles.

<u>Step 5</u> Now have:

If $k=0$, done — glue in disk get S_g.

If $k \neq 0$

Exercise III.16 Explain why attaching 1 twisted ~~1-handle~~ handle adjacent to an isolated linked pair is homeomorphic to 3 twisted 1-handles.

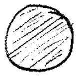


So, applying g times get

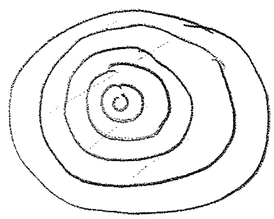
Now attach a disk and you have M_{k+2g}. $\square$

Surfaces & knots:

Suppose $K \subset S^3$ (or $\mathbb{R}^3$) is a knot. The genus of K, denoted $g(K)$, is the smallest $g \geq 0$ such that $K = \partial S$ with $S \subset S^3$ a surface homeomorphic to $S_{g,1}$

EX  unknot

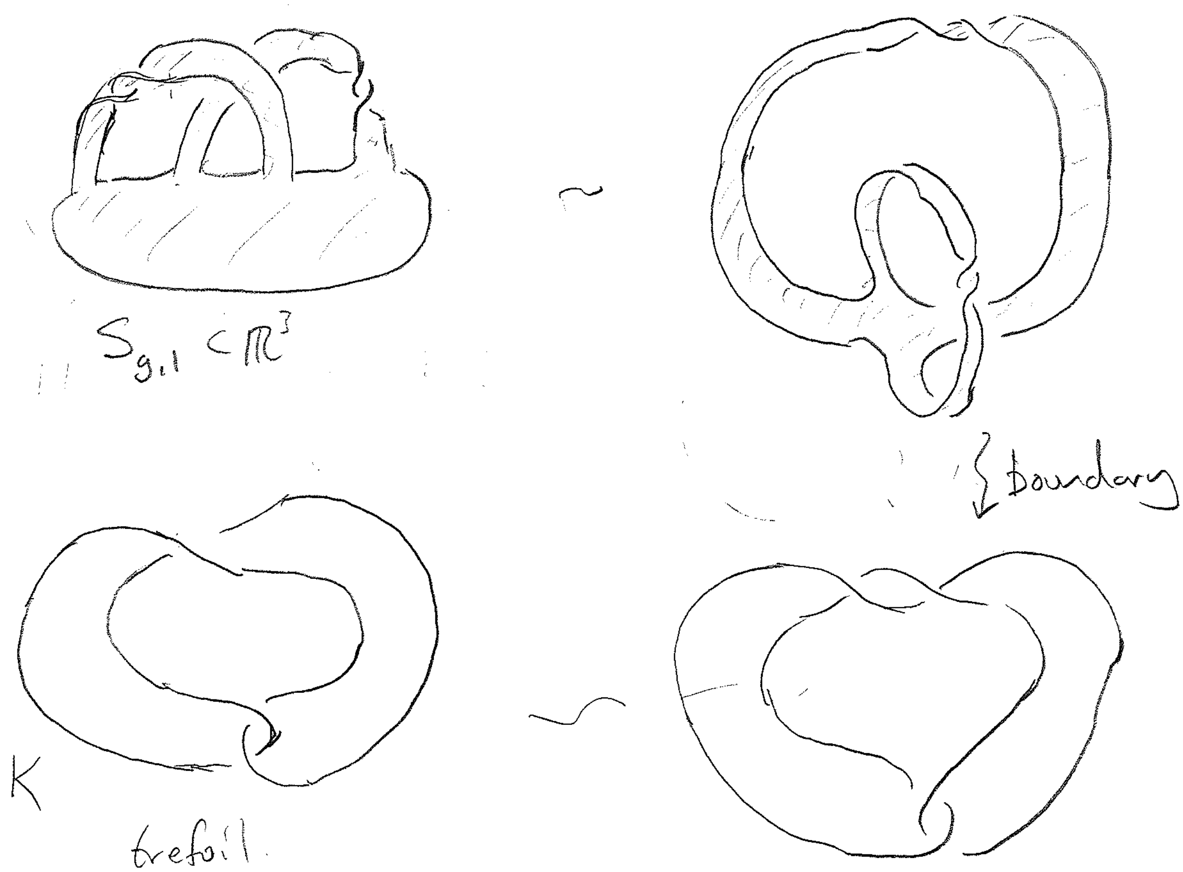
Fact: $g(K) = 0$ iff $K = O$. — clear: use the disk to shrink the knot to a small unknotted circle:



- We actually use polygonal surfaces in $\mathbb{R}^3 \in S^3$ to avoid pathologies.

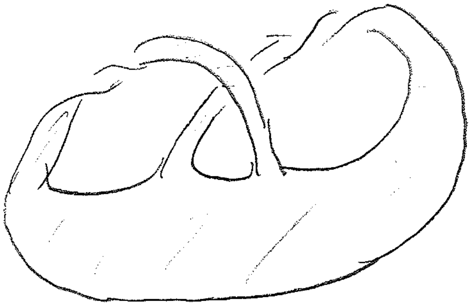
* Alexander Horned Sphere * •youtube•

Ex



$g(K) \neq 0$ (since $K \neq O$), so $g(K) = 1$.

similiar picture for fig 8:



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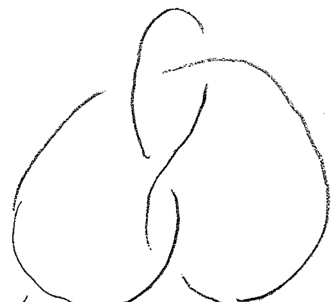
boundary



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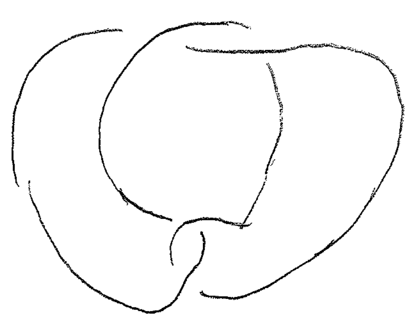

K_8

Fig 8. so $g(K_8)=1$.

<u>Theorem III.31</u> Every link $L \subset \mathbb{R}^3 (\subset S^3)$ is the boundary of a compact orientable surface $S \cong S_{g,k}$ w/ $k=|L|$.

<u>proof</u> (basically) <u>Seifert's Algorithm</u>

<u>Step 1</u> orient the link

<u>Step 2</u> consider a nice projection

<u>Step 3</u> Use the orientation resolve all crossings to produce a collection of <u>oriented</u> circles