III The Link group and some topology

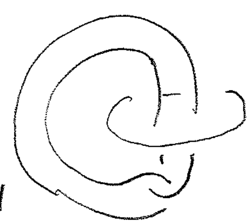
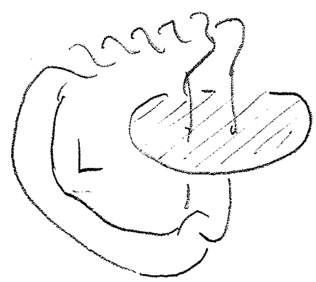
We now switch gears, turning our attention to topology. In particular the space $\mathbb{R}^3 \setminus L$ (or $S^3 \setminus L$), when L is a link.

If $L \sim L'$, then we have a homeomorphism
$$f : \mathbb{R}^3 \longrightarrow \mathbb{R}^3 \text{ with } f(L) = L'.$$
This f restricts to a homeomorphism $\hat{f} : \mathbb{R}^3 \setminus L \longrightarrow \mathbb{R}^3 \setminus L'$.

Therefore, quantities depending only on the homeomorphism type of $\mathbb{R}^3 - L$ become invariants of L.

A deep result of the late 1980's by Gordon and Luecke is that if K & K' are knots & $\mathbb{R}^3 \setminus K$ and $\mathbb{R}^3 \setminus K'$ are homeomorphic, then either $K \sim K'$ or $K \sim \overline{K}'$.

False for links:

W  and  have homeomorphic complements,

L

to see this "cut open" along disk, twist, and reglue

—more Later on this—