Here I'll try to explain why computing integrals of absolute values of functions requires special care. Before doing that, let's recall one way to compute definite integrals of absolute values of functions, that is when the integrand has the form $|f(x)|$. We do this by dividing the domain up into intervals on which $f(x) \geq 0$ and intervals on which $f(x) \leq 0$. Then adding up the results.

The example discussed in class today is:

$$\int_0^3 |2x^2 - 8| dx.$$

On the interval $[0, 3]$, we have $2x^2 - 8 \leq 0$ when $x \in [0, 2]$ and $2x^2 - 8 \geq 0$ when $x \in [2, 3]$. Alternatively, we can write

$$|2x^2 - 8| = \begin{cases} 8 - 2x^2 & \text{for } x \in [0, 2] \\ 2x^2 - 8 & \text{for } x \in [2, 3] \end{cases}$$

So, the definite integral is computed as

$$\begin{aligned} \int_0^3 |2x^2 - 8| dx &= \int_0^2 8 - 2x^2 dx + \int_2^3 2x^2 - 8 dx \\ &= [8x - 2x^3/3]_0^2 + [2x^3/3 - 8x]_2^3 \\ &= (16 - 16/3) + ((18 - 24) - (16/3 - 16)) \\ &= 26 - 32/3 = 46/3. \end{aligned}$$

Now, if we would like to explicitly compute the antiderivative

$$\int |2x^2 - 8| dx$$

we proceed as follows.

The generic antiderivative on the interval $[0, 2]$ is given by $8x - 2x^2/3 + C$ while the generic antiderivative on $[2, 3]$ is given by $2x^3/3 - 8x + C'$, for constants C and C'. On the interval $[0, 3]$, the antiderivative is given by

$$\int |2x^2 - 8| dx = \begin{cases} 8x - 2x^3/3 + C & \text{for } x \in [0, 2] \\ 2x^3/3 - 8x + C' & \text{for } x \in [2, 3] \end{cases}$$

However, the constants C and C' cannot be chosen arbitrarily: these antiderivatives must agree when $x = 2$. Therefore, we have

$$\begin{aligned} 8(2) - 2(2)^3/3 + C &= 2(2)^3/3 - 8(2) + C' \\ 16 - 16/3 + C &= 16/3 - 16 + C' \\ C' &= 32 - 32/3 + C \\ C' &= 64/3 + C \end{aligned}$$

So, we can write the general antiderivative as

$$\int |2x^2 - 8| dx = \begin{cases} 8x - 2x^3/3 + C & \text{for } x \in [0, 2] \\ 2x^3/3 - 8x + 64/3 + C & \text{for } x \in [2, 3] \end{cases}$$

This illustrates the reason for doing the computation on the pieces, as we described above. I hope this is helpful.