Math 520: Fixes

October 3, 2006

1 Regular value Theorem

In the statement of the regular value theorem, we should assume the preimage is **not** empty. The empty set is not a manifold.

Note that this does not affect the **definition** of a regular value (which still allows the preimage to be empty).

2 Transversality Theorem

Assume

$$F : M \times S \to N \supset Z$$

and

$$F \pitchfork Z \text{ with } W = F^{-1}(Z)$$

Set

$$\pi : M \times S \to S$$

and assume s a regular value of $\pi|_W$.

Claim: $f_s = F|_{M\times\{s\}}$ is transverse to Z.

Proof. Let $(m, s) \in M \times S$ so that $F(m, s) \in Z$. We note that

$$T_{(m,s)}(M \times \{s\}) = \ker(d\pi_{(m,s)})$$

Since s is a regular value of $\pi|_W$, we have

$$d(\pi|_W)_{(m,s)}(T_{(m,s)}W) = T_s(S)$$

and therefore,

$$T_{(m,s)}(M \times \{s\}) + T_{(m,s)}W = T_{(m,s)}(M \times S)$$

We need to show

$$dF_{(m,s)}(T_{(m,s)}(M \times \{s\})) + T_{F(m,s)}(Z) = T_{F(m,s)}(N)$$

Since $F \pitchfork Z$ we have

$$\begin{aligned}
T_{F(m,s)}N &= dF_{(m,s)}(T_{(m,s)}(M \times S)) + T_{F(m,s)}(Z) \\
&= dF_{(m,s)}(T_{(m,s)}(M \times \{s\}) + T_{(m,s)}(W)) + T_{F(m,s)}(Z) \\
&= dF_{(m,s)}(T_{(m,s)}(M \times \{s\})) + dF_{(m,s)}(T_{(m,s)}(W)) + T_{F(m,s)}(Z) \\
&= dF_{(m,s)}(T_{(m,s)}(M \times \{s\})) + dF_{(m,s)}(dF^{-1}_{(m,s)}(T_{F(m,s)}(Z))) + T_{F(m,s)}(Z) \\
&= dF_{(m,s)}(T_{(m,s)}(M \times \{s\})) + T_{F(m,s)}(Z)
\end{aligned}$$

as required. The last equality follows from the fact that

$$dF_{(m,s)}(dF^{-1}_{(m,s)}(T_{F(m,s)}(Z))) \subset T_{F(m,s)}(Z)$$

and therefore

$$dF_{(m,s)}(dF^{-1}_{(m,s)}(T_{F(m,s)}(Z))) + T_{F(m,s)}(Z) = T_{F(m,s)}(Z)$$

In class, I mistakenly wrote the containment above as an equality, which need not be true.