"$\epsilon - \delta$ implies continuity."

Suppose $f : \mathbb{R}^n \to \mathbb{R}^m$ has the property that for every $x \in \mathbb{R}^n$ and $\epsilon > 0$ there exists $\delta > 0$ so that if $|x' - x| < \delta$, then $|f(x) - f(x')| < \epsilon$. If we let $B(x, r)$ denote the ball of radius r about x, then this is the same as saying that

$$f(B(x, \delta)) \subset B(f(x), \epsilon).$$

Now let U be any open set. This is a union

$$U = \bigcup_{z \in U} B(z, \epsilon_z)$$

for some $\epsilon_z > 0$, one for every $z \in U$. By assumption, for every $z \in U$ and $x \in f^{-1}(z)$ there exists δ_x so that

$$f(B(x, \delta_x)) \subset B(z, \epsilon_z).$$

It follows that

$$f^{-1}(U) = \bigcup_{x \in f^{-1}(U)} B(x, \delta_x)$$

which is open.