# Differential Geometry: Problem set 3

September 10, 2006

Due Wednesday September 20

Read GP §1.3–1.8. Do problems:
§1.3: 5, 9;
§1.4: 2, 11;
§1.5: 7, 8;
§1.7: 6;
§1.8: 4.

1. Decide where the map
$$F : \mathbb{R}^2 \to \mathbb{R}^3$$
given by
$$F(x, y) = (x\cos(y), x\sin(y), x)$$
is an immersion.

2. (a.) Check that
$$\pi : \mathbb{R}^{n+1} \setminus \{0\} \to \mathbb{RP}^n$$
is a submersion.

   (b.) Check that the restriction of $\pi$ to $S^n \subset \mathbb{R}^{n+1} \setminus \{0\}$ is local diffeomorphism.

3. (a.) Check that
$$\pi : \mathbb{C}^{n+1} \setminus \{0\} \to \mathbb{CP}^n$$
is a submersion.

   (b.) Check that the restriction of $\pi$ to $S^{2n+1} \subset \mathbb{C}^{n+1} \setminus \{0\}$ is a submersion.

   (c.) Check that the fibers of $\pi|_{S^{2n+1}}$ are circles. Find an action of the Lie group(!) $S^1$ on $S^{2n+1}$ for which the orbits are precisely the fibers of $\pi|_{S^{2n+1}}$.

4. Verify that a submersion is an open mapping.

5. Verify that the projection $\pi : TM \to M$ from the tangent bundle of a smooth manifold $M$ to $M$ is a submersion.