Differentiable Manifolds: Problem set 3

Due Monday September 22

Read GP §1.3–1.5. Do problems:
§1.3: 5, 6a,b, 9; §1.4: 1, 2, 7, 11;

Extra problems.
1. Decide where the map
   \[ F : \mathbb{R}^2 \to \mathbb{R}^3 \]
   given by
   \[ F(x, y) = (x \cos(y), x \sin(y), x) \]
   is an immersion.

2. (a.) Check that
   \[ \pi : \mathbb{R}^{n+1} \setminus \{0\} \to \mathbb{R}^n \]
   is a submersion.
   (b.) Check that the restriction of \( \pi \) to \( S^n \subset \mathbb{R}^{n+1} \setminus \{0\} \) is local diffeomorphism.

3. (a.) Check that
   \[ \pi : \mathbb{C}^{n+1} \setminus \{0\} \to \mathbb{C}^n \]
   is a submersion.
   (b.) Check that the restriction of \( \pi \) to \( S^{2n+1} \subset \mathbb{C}^{n+1} \setminus \{0\} \) is a submersion.
   (c.) Check that the fibers of \( \pi|_{S^{2n+1}} \) are circles. Find an action of the Lie group(!) \( S^1 \) on \( S^{2n+1} \) for which the orbits are precisely the fibers of \( \pi|_{S^{2n+1}} \).

4. If \( M \) is a smooth manifold, verify that the projection \( \pi : TM \to M \) from the tangent bundle to the manifold is a submersion.