Instructions: You have 3 hours to complete this exam. Calculators, books, notes, and suchlike aids are not permitted. There are 80 points available and not all problems are weighted equally. Several problems are multiple choice questions. For non-multiple choice problems, show work that justifies your answer as in those problems credit will not be given for correct answers without proper justification. Work written outside of the space provided for a problem will not be graded. The last page of the exam contains a table of trigonometric identities.

Do not open exam until instructed.

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1. (a) (3 points) Consider a line $L$ with parameterization $\mathbf{r}(t) = \langle t - 1, 2t - 2, 3t - 3 \rangle$. Find symmetric equations for $L$.

Symmetric equations for $L$ are (circle one):

(A) $x - 3 = \frac{y - 2}{2} = \frac{z - 3}{3}$;  (B) $x = 2y = 3z$;

(C) $x + 1 = \frac{y + 2}{2} = \frac{z + 3}{3}$;  (D) $x - 2 = \frac{y - 2}{2} = \frac{z - 2}{3}$;

(b) (3 points) Consider the plane in $\mathbb{R}^3$ through the points $(1, 1, 1), (0, 2, 1), (-2, 2, 2)$. Find a unit normal vector $\mathbf{n}$ to the plane.

Unit normal vector is (circle one):

(A) $\mathbf{n} = \frac{1}{\sqrt{6}}(1, -1, 2)$;  (B) $\mathbf{n} = \frac{1}{\sqrt{6}}(1, 1, 2)$;

(C) $\mathbf{n} = \frac{1}{\sqrt{5}}(0, 2, 1)$;  (D) $\mathbf{n} = \frac{1}{\sqrt{9}}(-1, 2, 2)$;
2. (a) **(3 points)** Find the limit:

\[ L = \lim_{(x,y) \to (0,0)} \frac{3x^2y}{x^2 + 8y^2}. \]

(circle one): (A) \( L = 0 \); (B) \( L = 1 \); (C) \( L = \frac{3}{8} \); (D) DNE;

(b) **(2 points)** Find the limit

\[ L = \lim_{(x,y) \to (0,0)} \frac{xy}{\sqrt{4 + xy} - 2}. \]

(circle one): (A) \( L = 0 \); (B) \( L = 2 \); (C) \( L = 4 \); (D) DNE;

(c) **(3 points)** Find the limit

\[ L = \lim_{(x,y) \to (0,0)} \frac{x^4 - 10y^2}{x^2 + 5y^2}. \]

(circle one): (A) \( L = 0 \); (B) \( L = 10 \); (C) \( L = \frac{1}{5} \); (D) DNE;
3. (a) (3 points) Let \( f(x, y) = 3xe^{xy} \). Use the linearization of \( f(x, y) \) at \((1, 0)\) to find an approximate value for \( f(1.1, 0.1) \)

Approximate value is (circle one):

- (A) \( f(1.1, 0.1) \approx 3 \)
- (B) \( f(1.1, 0.1) \approx 3.6 \)
- (C) \( f(1.1, 0.1) \approx 0 \)
- (D) \( f(1.1, 0.1) \approx 1.2 \)

(b) (3 points) Let \( f(x, y) = 4(x - 1)^2 + 4(y + 3)^2 + 2 \). Find an equation for the tangent plane to the graph of \( f(x, y) \) at \((x, y) = (2, -2)\).

Equation for the tangent plane is (circle one):

- (A) \( z - 10 = 8(x - 1) + 8(y + 3) \)
- (B) \( z + 10 = 4(x - 2) + 4(y + 2) \)
- (C) \( z - 10 = 8(x - 2) + 8(y + 2) \)
- (D) DNE
4. Let \( f(x, y) = x^2 - 2x^3 + x^4 + y^2 - 2xy^2 - y^4 \).

(a) (3 points) Which of the following statements is true?

(A) \( f(x, y) \) has a local minimum at \((0, 0)\).
(B) \( f(x, y) \) has a local maximum at \((0, 0)\).
(C) \( f(x, y) \) has a saddle point at \((0, 0)\).
(D) \((0, 0)\) is not a critical point of \( f(x, y) \).

(circle one): (A) is true; (B) is true; (C) is true; (D) is true;

(b) (3 points) Which of the following statements is true?

(A) \( f(x, y) \) has a local minimum at \((1, 0)\).
(B) \( f(x, y) \) has a local maximum at \((1, 0)\).
(C) \( f(x, y) \) has a saddle point at \((1, 0)\).
(D) \((1, 0)\) is not a critical point of \( f(x, y) \).

(circle one): (A) is true; (B) is true; (C) is true; (D) is true;
5. (a) (3 points) Let $C$ be the part of the ellipse $x^2/4 + y^2/9 = 1$ in the upper-half plane ($y \geq 0$) from $(-2, 0)$ to $(2, 0)$. Give a parameterization $\vec{r}(t)$ of $C$.

\[
\vec{r}(t) = \langle \quad, \quad \rangle, \quad \text{with } t \in [\quad, \quad]
\]

(b) (3 points) Let $C$ be the circle with center $(0, 0)$ and radius 2. Find a parameterization $\vec{R}(t)$ of the tangent line to $C$ at the point $(1, \sqrt{3})$.

\[
\vec{R}(t) = \langle \quad, \quad \rangle
\]
6. \textbf{(6 points)} Consider the function $f(x, y, z) = 4x + 4y + 2z$. Find the maximum value of $f$ on the surface $2x^2 + 2y^2 + 2z^2 = 18$.

The maximum value of $f$ is: _______
7. (a) (3 points) Let \( \vec{F} = \langle 3 + 2xy, x^2 - 3y^2 \rangle \) and let \( C \) be the curve with parameterization \( \vec{r}(t) = \langle e^t \sin(t), e^t \cos(t) \rangle \), \( 0 \leq t \leq \pi \). Find the line integral:

\[
\int_C \vec{F} \cdot d\vec{r}.
\]

\[
\int_C \vec{F} \cdot d\vec{r} = \text{(circle one)} \quad (A) 0; \quad (B) e^{3\pi} - 1; \quad (C) e^{3\pi} + 1; \quad (D) \text{DNE};
\]

(b) (3 points) Let \( \vec{F} = \langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \rangle \) and let \( C \) be the circle with radius 1 and center \((0, 0)\), and \( D \) the circle with radius 1 and center \((5, 0)\), both with the positive orientation. Consider the two integrals

\[
I_C = \oint_C \vec{F} \cdot d\vec{r}, \quad I_D = \oint_D \vec{F} \cdot d\vec{r}
\]

Which of the following statements is true?

(A) \( I_C \) and \( I_D \) both are zero.
(B) \( I_C = 0 \) and \( I_D = 2\pi \).
(C) \( I_C = 2\pi \) and \( I_D = 0 \).
(D) \( I_C = I_D = 2\pi \).

(circle one): (A) is true; (B) is true; (C) is true; (D) is true;
8. (a) (3 points) Find the area of the region $D$ in the plane bounded by the curves $x = y^2 - 3$ and $2y = x$.

$$\text{Area of } D \text{ is (circle one): (A) } \frac{29}{3}; \quad \text{(B) } \frac{30}{3}; \quad \text{(C) } \frac{31}{3}; \quad \text{(D) } \frac{32}{3};$$

(b) (3 points) Consider a solid cylinder $x^2 + y^2 \leq 4$ with density of mass $\rho(x, y, z) = e^{x^2 + y^2}$. What is the total mass $M$ of the portion $R$ of the cylinder with $x, y, z \geq 0$ that lies below the plane $z = 3$?

$$M \text{ is (circle one): (A) } \pi \frac{(e^2-1)}{4}; \quad \text{(B) } \frac{3\pi(e^4-1)}{4}; \quad \text{(C) } \frac{3\pi(e^2-1)}{4}; \quad \text{(D) } \frac{\pi(e^4-1)}{4};$$
9. *(6 points)* Let $C$ be the curve in the plane with parameterization $\vec{r}(t) = \langle 2\cos(t), -2\sin t \rangle$, where $t \in [0, 2\pi]$. Find the line integral $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F}(x, y) = \langle 2xy, x^2 + 2y + 2x \rangle$, using a method of your choice.

$$\int_C \vec{F} \cdot d\vec{r} = \square$$
10. (a) (3 points) Which one of the following statements makes sense and is true for any vector field $\vec{F}$ in $\mathbb{R}^3$ whose components have continuous second partial derivatives?

(A) $\text{grad}(\text{curl}(\vec{F})) = \vec{0}$.
(B) $\text{div}(\text{curl}(\vec{F})) = 0$.
(C) $\text{div}(\text{grad}(\vec{F})) = 0$.
(D) $\text{curl}(\text{curl}(\vec{F})) = \vec{0}$.

(circle one): (A) is true; (B) is true; (C) is true; (D) is true;

(b) (3 points) Which one of the following statements makes sense and is true for a vector field $\vec{F}$ defined in $\mathbb{R}^3$ whose components have continuous partial derivatives?

(A) If $\text{div}(\vec{F}) = 0$ then $\vec{F}$ is conservative.
(B) If $\vec{F}$ is conservative then $\text{div}(\vec{F}) = 0$.
(C) If $\text{curl}(\vec{F}) = \vec{0}$ then $\vec{F}$ is conservative.
(D) If $\vec{F}$ is conservative then $\text{grad}(\vec{F}) = \vec{0}$.

(circle one): (A) is true; (B) is true; (C) is true; (D) is true;
11. Let \( D = \{ (u, v) : 0 \leq u \leq 1, 0 \leq v \leq 1 \} \) be the unit square and let \( R \subset \mathbb{R}^2 \) be the parallelogram with vertices \((0,0), (1,-1), (2,2)\) and \((3,1)\).

(a) \(\text{(3 points)}\) Find a transformation \( T : \mathbb{R}^2 \to \mathbb{R}^2 \) such that \( T(D) = R \).

\[
T(u, v) = ( , )
\]

(b) \(\text{(3 points)}\) Evaluate the integral:

\[
\iint_R xy \, dA.
\]

(Hint: It maybe easier to use the change of variables \((x, y) = T(u, v)\) that you found in (a)).

\[
\iint_R xy \, dA =
\]
12. **(6 points)** In $\mathbb{R}^3$ consider the vector field $\vec{G} = \langle z^2, x^2, y^2 \rangle$. Find the flux:

$$\iint_S \vec{G} \cdot \vec{n} \, dS,$$

where $S \subset \mathbb{R}^3$ is a surface whose boundary is the ellipse $x^2 + 2y^2 = 2$ in the $xy$-plane. Use the orientation on $S$ for which the boundary is oriented clockwise.

(HINT: One has $\vec{G} = \text{curl}(\vec{F})$, where $\vec{F} = \langle x^2 z, y^2 x, z^2 y \rangle$.)

$$\iint_S \vec{G} \cdot \vec{n} \, dS =$$
13. (6 points) In $\mathbb{R}^3$ consider the vector field $\vec{F} = \langle xz - x, ze^x, z \rangle$. Find the flux:

$$\iint_S \vec{F} \cdot \vec{n} \, dS,$$

where $S \subset \mathbb{R}^3$ is the boundary of the half-ball $R = \{(x, y, z) : x^2 + y^2 + z^2 \leq 2, z \geq 0\}$, oriented with the outward pointing normal.

$$\iint_S \vec{F} \cdot \vec{n} \, dS =$$
**Trigonometric Identities**

\[
\sin(\theta + \phi) = \sin \theta \cos \phi + \cos \theta \sin \phi \\
\cos(\theta + \phi) = \cos \theta \cos \phi - \sin \theta \sin \phi \\
\tan(\theta + \phi) = \frac{\tan(\theta) + \tan(\phi)}{1 - \tan \theta \tan \phi} \\
\sin(\theta - \phi) = \sin \theta \cos \phi - \cos \theta \sin \phi \\
\cos(\theta - \phi) = \cos \theta \cos \phi + \sin \theta \sin \phi \\
\tan(\theta - \phi) = \frac{\tan(\theta) - \tan(\phi)}{1 + \tan \theta \tan \phi} \\
\sin(\theta) \sin(\phi) = \frac{\cos(\theta - \phi) - \cos(\theta + \phi)}{2} \\
\cos(\theta) \cos(\phi) = \frac{\cos(\theta - \phi) + \cos(\theta + \phi)}{2} \\
\sin(\theta) \cos(\phi) = \frac{\sin(\phi + \theta) - \sin(\phi - \theta)}{2} \\
\sin(2\theta) = 2 \sin \theta \cos \theta \\
\cos(2\theta) = \cos^2 \theta - \sin^2 \theta \\
\tan(2\theta) = \frac{2 \tan \theta}{1 - \tan^2 \theta} \\
\sin \theta = \pm \sqrt{\frac{1 - \cos(2\theta)}{2}} \\
\cos \theta = \pm \sqrt{\frac{1 + \cos(2\theta)}{2}} \\
\tan \theta = \frac{\sin(2\theta)}{1 + \cos(2\theta)}
\]