A continuous vector field $\mathbf{F}$ on an open subset $D$ in $\mathbb{R}^n$ is conservative if and only if $\mathbf{F} = \nabla f$.

True or False?

A True.

B False.

C I don’t know.
For any function $f$ on an open subset $D$ in $\mathbb{R}^3$ with continuous second order partial derivatives, $\text{curl}(\nabla f) = 0$.

True or False?

A True.

B False.

C I don’t know.
Conservative vector fields, revisited

**Theorem A.** A continuous vector field $\mathbf{F}$ on an open subset $D$ in $\mathbb{R}^n$ is conservative if and only if $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$ for every closed curve $C$ in $D$.

**Theorem B.** A vector field $\mathbf{F}$ on $\mathbb{R}^3$ with continuous first order partial derivatives is conservative if and only if $\text{curl}(\mathbf{F}) = 0$. 
The vector field \( \mathbf{F} = \frac{1}{(x^2+y^2)^{3/2}} \langle -y, x, 0 \rangle \) is conservative.

True or False?

A True.

B False.

C I don’t know.
For any vector field $\mathbf{F}$ on an open subset $D$ in $\mathbb{R}^3$ with continuous second order partial derivatives, $\text{div} (\text{curl} (\mathbf{F})) = 0$.

True or False?

A True.
B False.
C I don’t know.
Theorem C. A continuous vector field $\mathbf{F}$ on an open set $D$ in $\mathbb{R}^3$ is the curl of some other vector field if and only if
$$\int\int_S \mathbf{F} \cdot d\mathbf{S} = 0$$
for every oriented surface $S$ in $D$ that bounds a region $R$ in $\mathbb{R}^3$.

Theorem D. A vector field $\mathbf{F}$ on $\mathbb{R}^3$ with continuous first order partial derivatives is curl of another vector field if and only if $\text{div}(\mathbf{F}) = 0$. 