**Projections, distances, and planes.**

1. Let \( \mathbf{a} = \mathbf{i} + \mathbf{j} \) and \( \mathbf{b} = 2\mathbf{i} - \mathbf{j} \).

   (a) Calculate \( \text{proj}_b \mathbf{a} \) and draw a picture of it together with \( \mathbf{a} \) and \( \mathbf{b} \).

   (b) The orthogonal complement of the vector \( \mathbf{a} \) with respect to \( \mathbf{b} \) is defined by

   \[ \text{orth}_b \mathbf{a} = \mathbf{a} - \text{proj}_b \mathbf{a}. \]

   Calculate \( \text{orth}_b \mathbf{a} \) and draw two copies of it in your picture from part (a), one based at \( \mathbf{0} \) and the other at \( \text{proj}_b \mathbf{a} \).

   (c) Check that \( \text{orth}_b \mathbf{a} \) calculated in (b) is orthogonal to \( \text{proj}_b \mathbf{a} \) calculated in (a).

   (d) Find the distance of the point \((1, 1)\) from the line \((x, y) = t(2, -1)\). Hint: relate this to your picture.

2. Let \( \mathbf{a} \) and \( \mathbf{b} \) be vectors in \( \mathbb{R}^n \). Use the definitions of \( \text{proj}_b \mathbf{a} \) and \( \text{orth}_b \mathbf{a} \) to show that \( \text{orth}_b \mathbf{a} \) is always orthogonal to \( \text{proj}_b \mathbf{a} \).

3. Find the distance between the point \( P(3, 4, -1) \) and the line \( \mathbf{l}(t) = (2, 3, -2) + t(1, -1, 1) \). Hint: Consider a vector starting at some point on the line and ending at \( P \), and connect this to what you learned in Problem 1.

4. Consider the equation of the plane \( x + 2y + 3z = 12 \).

   (a) Find a normal vector \( \mathbf{n} \) to the plane. (Just look at the equation!)

   (b) Find where the \( x \), \( y \), and \( z \)-axes intersect the plane. Using this information, sketch the portion of the plane in the first octant where \( x \geq 0, y \geq 0, z \geq 0 \).

   (c) Using the points in part (b), find two non-parallel vectors that are parallel to the plane.

   (d) Using the dot product to check that the vectors you found in (c) are really orthogonal to \( \mathbf{n} \).

   (e) Pick another normal vector \( \mathbf{n}' \) to the plane and one of the points from (b). Use these to find an alternative equation for the plane. Compare this new equation to \( x + 2y + 3z = 12 \). How are these two equations related? Is it clear that they describe the same set of points \((x, y, z)\) in \( \mathbb{R}^3 \)?

5. *The Triangle Inequality.* Let \( \mathbf{a} \) and \( \mathbf{b} \) be any vectors in \( \mathbb{R}^n \). The triangle inequality states that \( |\mathbf{a} + \mathbf{b}| \leq |\mathbf{a}| + |\mathbf{b}|. \)

   (a) Give a geometric interpretation of the triangle inequality. (E.g. draw a picture in \( \mathbb{R}^2 \) or \( \mathbb{R}^3 \) that represents this inequality.)

   (b) Use what we know about the dot product to explain why \( |\mathbf{a} \cdot \mathbf{b}| \leq |\mathbf{a}| |\mathbf{b}| \). This is called the Cauchy-Schwarz inequality.

   (c) Use part (b) to justify the triangle inequality. Hint: Start with the fact that \( |\mathbf{a} + \mathbf{b}|^2 = (\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} + \mathbf{b}) \) and then use properties of the dot product and the Cauchy-Schwarz inequality to manipulate the right-hand side into looking like \( |\mathbf{a}|^2 + |\mathbf{b}|^2 \).