Homework Assignment 12
Math 417, Fall 2015

Read: Goodman sections 6.2-6.4

Problems (from Goodman).

1. 6.2.18

2. Prove that $x^2 + 1$ is irreducible in the ring of polynomials $\mathbb{Z}_7[x]$ over the field $\mathbb{Z}_7$.

3. Find an isomorphism from the quotient field $\mathbb{Q}[x]/(x^2 - 2)$ to the field $\mathbb{Q}(\sqrt{2}) = \{a + b\sqrt{2} \mid a, b \in \mathbb{Q}\}$.

4. The polynomial $p(x) = x^4 + x^3 + x^2 + x + 1$ is irreducible in the ring of polynomials $\mathbb{Q}[x]$ (you do not need to prove this). Let $K = \mathbb{Q}[x]/(p)$ be the quotient field. Prove that $p$ has 4 distinct roots in $K$. Hint: Let $\alpha \in K$ be root of $p$. Prove that $\alpha^5 = 1$ and then prove that $\alpha^2, \alpha^3, \alpha^4$ are also roots of $p$. Finally, prove that they are all distinct, meaning that for $1 \leq i < j \leq 4$, if $\alpha^i = \alpha^j$, then $i = j$. For this last part, you may want to use the fact that $p(x)$ is irreducible in $\mathbb{Q}[x]$.

5. 6.4.10

Due: Wednesday December 9, beginning of class.