Topologies on $\{1, 2, 3\}$

One way to enumerate the **29 topologies** on $\{1, 2, 3\}$ is by keeping track of the number of 1-point sets in your topology. Let's write 12 for the set $\{1, 2\}$ to save on notation.

For a topology with no one-point sets you get:

123,$\emptyset$
123,$\emptyset$,12
123,$\emptyset$,13
123,$\emptyset$,23

Since we always have 123 and $\emptyset$, we won't list these, but just keep in mind that they are there...

For a topology with exactly one one-point set you get:

1
1,12
1,13
1,23
1,12,13
2
2,12
2,23
2,13
2,12,23
3
3,13
3,23
3,12
3,13,23

For a topology with two one-point sets you get:

1,2,12
1,2,12,13
1,2,12,23
1,3,13
1,3,13,12
1,3,13,23
2,3,23
2,3,23,13
2,3,23,12

Finally, if you have all one-point sets, you get the discrete topology:

1,2,3,12,13,23