Math 234BL1 Practice Exam I

1. Find the domain of the function \( f(x) = \frac{\sqrt{9 - x^2}}{2 - x} \).

2. Write an equation for the line that passes through \((4, 1)\) and perpendicular to the line \( x + 2y = 3 \).

3. Find the indicated limit if it exists or state that the limit does not exist.
   (a) \( \lim_{x \to \infty} \frac{x + 1}{x - 1} \)
   (b) \( \lim_{x \to 2} \frac{\sqrt{x + 7} - 3}{x - 2} \)
   (c) \( \lim_{x \to 1} \frac{2x^2 - x - 1}{x - 1} \)

4. Suppose that \( \lim_{x \to \infty} \frac{bx^2 + 4x + 1}{ax^2 + 3x + 1} = 2 \) and \( \lim_{x \to -\infty} \frac{-2x^2 + 3}{ax^2 + bx + 1} = -3 \). Find the values of \( a \) and \( b \).

5. Suppose that the function \( f(x) \) is defined by
   \[
   f(x) = \begin{cases} 
   2x + 2 & \text{if } x < 2 \\
   4 & \text{if } x \geq 2.
   \end{cases}
   \]
   (a) Find \( \lim_{x \to 2^-} f(x) \).
   (b) Find \( \lim_{x \to 2^+} f(x) \).
   (c) Is \( f(x) \) continuous at \( x = 2 \)? Why?

6. (a) State the definition of \( f'(x) \).
   (b) Use the the definition in (a) to compute \( f'(1) \) when \( f(x) = \frac{1}{3x + 2} \).

7. Find the equation of the tangent line to the graph of \( f(x) = \sqrt{x} + \frac{1}{x} \) for \( x = 1 \).

8. (20pts) Differentiate. (Do not simplify)
   (a) \( y = x^3(2x - 5)^2 \)
   (b) \( y = \frac{\sqrt{x^2 + 1}}{2x^2} \)

9. Find the first and the second derivatives.
   \[
   f(t) = (t^2 + 1)^7
   \]
10. When $x$ units of a certain commodity are produced, they can all be sold at a price of $p$ hundred dollars per unit, where

$$p = \frac{x}{x^2 + 1}.$$ 

(a) Express the revenue $R(x)$ as a function of $x$.

(b) At what rate is the revenue changing with respect to $x$ when 3 units are produced? Is revenue increasing or decreasing at this level of production?

11. A biologist models the effect of introducing a toxin to a bacterial colony by the function

$$P(t) = \frac{24t + 10}{t^2 + 1}$$

where $P$ is the population of the colony $t$ hours after the toxin is introduced. At what time does the population begin to decrease? By how much does the population increase before it begins to decline?