Math 234 Practice Exam II

1. Use the implicit differentiation to find $\frac{dy}{dx}$.
   (a) $\frac{x^2}{4} + \frac{y^2}{9} = 1$.
   (b) $xy^3 - x^3y = 1$.

2. Find the equation of the tangent line to the given curve at the specified point.
   $(x - 2)^2 - (y - 3)^2 = 5, \quad (5, 5)$.

3. Let $f(x) = x^3 - 3x + 1$.
   (a) Find all critical points of $f(x)$.
   (b) Identify the critical points as relative maximum, relative minimum or neither.
   (c) Find all inflections points of $f(x)$.
   (d) Sketch the graph of $f(x)$.

4. Let $f(x) = x + \frac{1}{x}, x > 0$. Find, if exits, absolute maximum and absolute minimum of $f(x)$. If it doesn’t exist, explain why.

5. Suppose that the demand function for a certain commodity is given by
   \[ q = \frac{100}{p + 3}. \]
   Find the elasticity $E(p)$ and determine if demand is elastic or inelastic (or neither) at $p = 10$.

6. Suppose that the price $p$ (in dollars) and the daily sales $x$ (in thousands of units) of a certain commodity satisfy the demand equation
   \[ 2p^2 + x = 240. \]
   Determine the rate at which sales are changing at a time when the price is 10 dollars and is falling at the rate of $.10$ per day.

7. word problems in Section 3.5. Do as many as you can.
Answer

1. (a) \( \frac{dy}{dx} = \frac{9x}{4y} \)
   (b) \( \frac{dy}{dx} = \frac{y^3 - 3x^2y}{3xy^2 - x^3} \)

2. \( y - 5 = \frac{3}{2}(x - 5) \)

3. (a) \((-1, 3), (1, -1)\) (b) \((-1, 3)\) : relative max, \((1, -1)\) : relative min. (c) \((0, 1)\)

4. absolute minimum : when \(x = 1\), maximum is \(f(1) = 2\)
   absolute maximum : does not exist because \(\lim_{x\to\infty} f(x) = \infty\).

5. \( E(p) = \frac{p}{p + 3} \). inelastic.

6. 4 thousands/day