Math 221, EL1 - Test #1 - September 27, 2012

Name: __________________________________________________________

Signature: ______________________________________________________

Circle your discussion section:   ED1 (9:00, Nathan)   ED2 (1:00, Dominic)
EDA (9:00, Sarka)           EDB (11:00, Sarka)     EDC (12:00, Sneha)
EDD (1:00, Sneha)           EDE (2:00, Panupong)  EDF (3:00, Panupong)

DO NOT OPEN EXAM UNTIL TOLD TO DO SO.

Time: 100 minutes. You may not use any books or calculator; you may use a single handwritten
notecard. There are 100 points possible. To get any credit, you must show your work. Unless indicated,
you do not need to simplify your answers. Partial credit will be based only on what is actually written on
the paper. All intermediate steps should be correct as written.

<table>
<thead>
<tr>
<th>problem number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>possible points</td>
<td>12</td>
<td>12</td>
<td>8</td>
<td>20</td>
<td>12</td>
<td>5</td>
<td>10</td>
<td>11</td>
<td>10</td>
</tr>
<tr>
<td>score</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1. (3 points per part) Compute $\frac{dy}{dx}$ in the following cases using any method we have studied so far. Your answer may include both x’s and y’s. You do not need to show work for some of these.

(a) $y = (3x + 9)^2$

(b) $y = e^3 + \pi e + 3\pi$

(c) $y = \tan^{-1}(x^4)$

(d) $y^3 + y = x^2 + x$
2. (4 points per part) Find $\frac{dy}{dx}$ in the following cases using any method we have studied so far. Your answer may include both x’s and y’s.

(a) $y = \cos^2(3x) - \sin^2(3x)$

(b) $y = \tan(e^{3x})$

(c) $y = f(g(x))$ where $f(x) = 9$ and $g(x) = \sin(e^x)$

3. (8 points) Follow the steps below to compute these two limits. You do not need to show work for the fill-in-the-blank questions.

(a) $\lim_{h \to 0} \frac{\sin((\pi + h)) - \sin(\pi)}{h}$ is the derivative of $y = \ldots$ at $x = \ldots$.

(b) Using your answer to part (a), find the value of the limit.

(c) $\lim_{h \to 0} \frac{e^{h} - e^{0}}{h}$ is the derivative of $y = \ldots$ at $x = \ldots$.

(d) Using your answer to part (c), find the value of the limit.
4. Let \( g(x) = |x^2 - 16| \).

(a) (8 points) Write a limit expression for \( g'(4) \) by using the definition of the derivative. Simplify your answer so that it does not contain any \( g \)'s or \( x \)'s.

(b) (3 points) Your answer to (a) should include a limit as \( h \) approaches 0. What do you get if you plug in \( h = -0.1 \)? Simplify your answer completely.

(c) (3 points) Your answer to (a) should include a limit as \( h \) approaches 0. What do you get if you plug in \( h = 0.1 \)? Simplify your answer completely.

(d) (6 points) Explain why \( g'(4) \) does or does not exist. Bring the answers to the previous two questions into your argument. (2-3 sentences)
5. Compute the following limits. Do not use L’Hopital’s Rule, which we have not yet studied. Show at least one step of work for every limit.

(a) (3 points) \( \lim_{x \to -1} \frac{x^2 - 2x - 3}{x-1} \)

(b) (3 points) \( \lim_{x \to -1} \frac{x^2 - 2x - 3}{x+1} \)

(c) (6 points) \( \lim_{x \to \infty} \sqrt{x^2 + 9x} - \sqrt{x^2 + 6x} \)
6. (5 points) The table below gives estimates for world population $P(t)$ in year $t$. $P(t)$ is given in millions of people.

<table>
<thead>
<tr>
<th>$t$</th>
<th>1750</th>
<th>1800</th>
<th>1850</th>
<th>1900</th>
<th>1950</th>
<th>2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(t)$</td>
<td>791</td>
<td>978</td>
<td>1262</td>
<td>1650</td>
<td>2519</td>
<td>6070</td>
</tr>
</tbody>
</table>

Try to use the Intermediate Value Theorem to show that at some point, there were exactly 900 million people on Earth. What goes wrong?

7. (10 points) Show that there is some value of $x$ such that $x^4 + x = 5$. Hint: You do not need to actually find the value of $x$. 
8. Below is a graph of height $h(t)$, given in meters, between $t = -3$ and $t = 9$. $t$ is given in hours.

(a) (5 points) Sketch a graph of the velocity $h'(t)$. For the purposes of this problem, do not worry about whether the derivative of the function is defined or not at the endpoints of the graph.

(b) (3 points) What is the average velocity from $t = 5$ to $t = 8$? Include the units.

(c) (3 points) What is the instantaneous velocity at $t = 4$? Include the units.
9. A falling object’s height at time $t$ is $f(t) = 100 - 10t^2$ meters where $t$ is in seconds.

(a) (1 point) Sketch a graph of $f(t)$.

(b) (1 point) Add a secant line to the graph connecting the points at $t = 1$ and $t = 1 + h$ where $h = 1.5$. Label it “(b)”.

(c) (2 points) What physical quantity does the slope of that line represent? Do not simply calculate the slope.

(d) (1 point) Draw a tangent line to the graph at $t = 1$. Label it “(d)”.

(e) (2 points) What physical quantity does the slope of that line represent? Do not simply calculate the slope.

(f) (1 point) Above you drew a secant line with $h = 1.5$. What value of $h$ would cause a secant line that was closer to the tangent line?

(g) (2 points) What happens as $h$ gets very close to 0? (1-2 sentences)