Levels of Mental Development in Geometry
P.H. Van Hiele, 1959

The work of 2 Dutch educators, Dina van Hiele-Geldof and Pierre van Hiele, is influencing the teaching of geometry. They were concerned about the difficulties their students were having with geometry, so they conducted research aimed at understanding children's levels of geometric thinking to determine the kinds of instruction that can best help children. Each Van Hiele level describes how children think about geometric concepts.

Level 0: Recognition or Visualization
Children at the visualization level think about shapes in terms of what they resemble. At this level, children are able to sort shapes into groups that look alike to them in some way.
Ex: "a mountain"

Level 1: Analysis (drawing and verbal skills)
Children at the analysis level think in terms of properties. They can list all of the properties of a figure but don't see any relationships between the properties, and don't realize that some properties imply others.

Level 2: Ordering or Informal Deductive (verbal skills)
Children at the informal deduction level not only think about properties but also are able to notice relationships within and between figures. At this level, children are able to
formulate meaningful definitions. At this level, children are also able to make and follow informal deductive arguments.

Ex: “All squares are rectangles, but not all rectangles are squares.”

Level 3: Deduction or Formal Deductive (logical skills)
Children at the formal deductive level think about relationships between properties of shapes and also understand relationships between axioms, definitions, theorems, corollaries, and postulates. They understand how to do a formal proof and understand why it is needed. For successful completion, the typical high school geometry course requires geometric understanding at the formal deduction level.

Level 4: Rigor (applied skills)
Children at the rigor level can think in terms of abstract mathematical systems. College mathematics majors and mathematicians are at this level.

In 1990, two researchers (Clements and Battista) hypothesized that a level exists that is below visualization. They called it “pre-recognition.” Children operating at this level wouldn’t be able to distinguish a three-sided figure from a four-sided figure.

Most elementary schoolers are at the visualization or analysis level; some middle-school children are at the informal deduction level. The 6th NAEP noted that most students (4th, 8th and 12th grades) appeared to be performing at the visualization level. It is desirable to have a child at the informal deduction level or above by the time he or she finishes middle school.

Comments on Levels:
• The levels are not age dependent, but rather, are related more to the experiences students have had.
• The levels are sequential; children must pass through the levels in order as their understanding increases (except for gifted children).
• To move from one level to the next, children need to have many experiences in which they are actively involved in exploring and communicating about their observations of shapes, properties, and relationships.
• For learning to take place, language must match the child’s level of understanding. If the language used is above the child’s level of thinking, the child may only be able to learn procedures and memorize without understanding.
• It is difficult for two people who are at different levels to communicate effectively. A teacher must realize that the meaning of many terms is different to the child than it is to the teacher and adjust his or her communication accordingly.

Ex: Given a “square”
A person at the visualization level will think of a CD case, because that is what a square looks like. A person at the informal deduction level thinks about the fact that a square has four congruent sides and four congruent angles and will know the properties of a square such as opposite sides parallel and the diagonals perpendicular bisectors.